

HAROLD E. EDGERTON

PAPERS

MC 25

Series III

Laboratory Notebooks

Number 3

Dated Feb. 15, 1930 to June 16, 1931

Massachusetts Institute of Technology

COMPUTATION BOOK

NAME

HAROLD E. FODOR

NUMBER

3

Course

7.01

Used from

FEB 15

1930, to

JUNE 16

1931.

Notebook # 3

Filming and Separation Record

___ unmounted photograph(s)

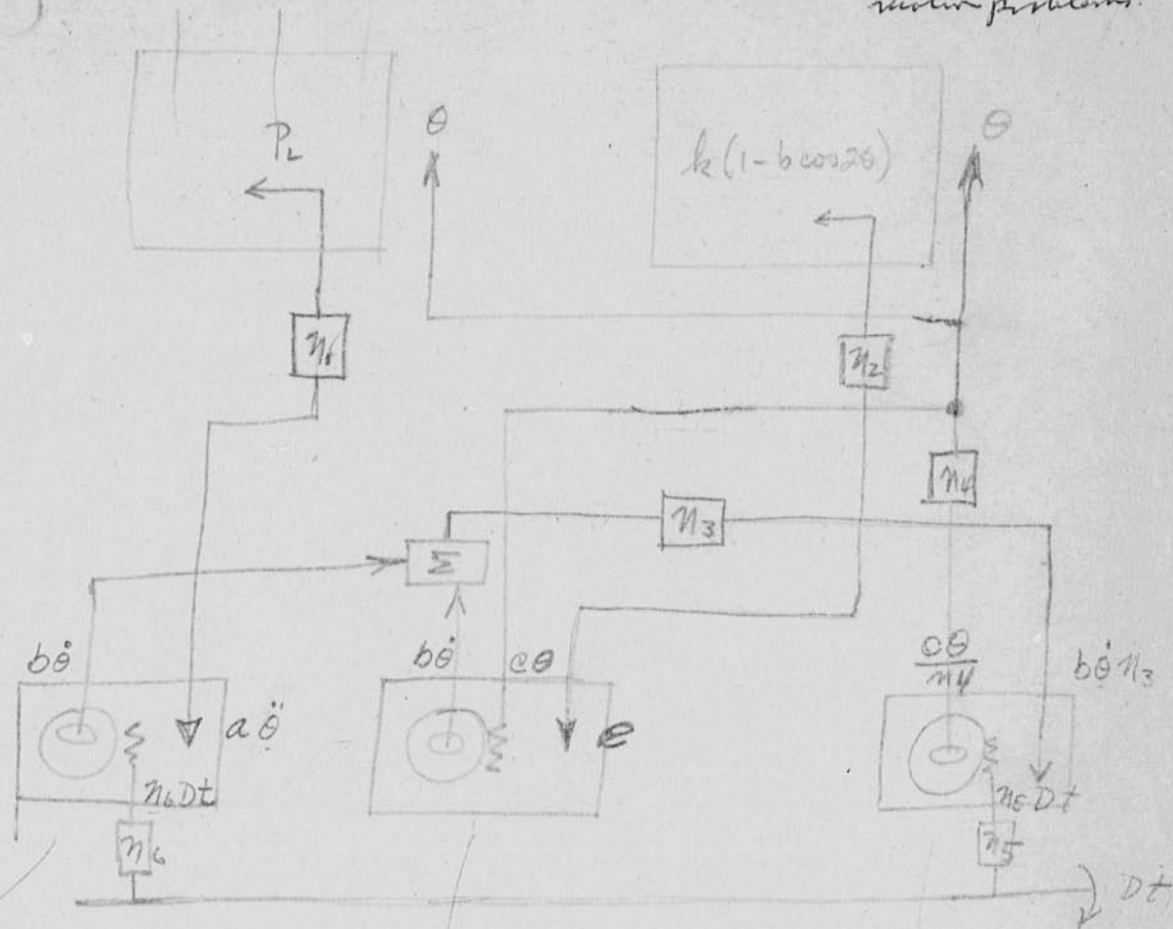
___ negative strip(s)

21 unmounted page(s)
(notes, drawings, letters, etc.)

was/were filmed where originally located between page ___ and ___.
inside front cover

Item(s) now housed in accompanying folder.

INTEGRAPH Solutions of similar motor problems.



$a\ddot{\theta} < 40 \text{ rev. } \checkmark$
 $e < 40 \text{ rev.}$
 $b\dot{\theta}\eta_3 < 40 \text{ rev.}$

$$b\dot{\theta} = \frac{1}{32} \int \frac{a\ddot{\theta} \eta_6 D dt}{b}$$

$$\frac{a \eta_6 D}{32 b} = 1$$

$$c\dot{\theta} = \frac{1}{32} \int \frac{c\theta}{b} \frac{d\theta}{dt} dt$$

$$\frac{ec}{32 b} = 1$$

$$\frac{c\theta}{\eta_4} = \frac{1}{32} \int \frac{b\dot{\theta} \eta_3 \eta_5 D dt}{c}$$

$$\frac{b \eta_4 \eta_3 \eta_5 D}{32 c} = 1$$

$$\dot{\theta}_{\max} = 20 \text{ units elect degs / rev. unit.}$$

$$b n_3 20 < 40 \text{ rev.}$$

$$b < \frac{40}{n_3 20} = \frac{2}{n_3}$$

$$\text{Let } n_3 = \frac{1}{4}$$

$$b < \frac{2}{\frac{1}{4}} = 8$$

$$(1) \text{ Let } n_6 = n_5 = 1$$

$$\frac{a n_6 D}{32 \times 8} = 1$$

(2)

$$\frac{e.c}{32 \times 8} = 1$$

(3)

$$\frac{8 n_4 n_5 D}{32 \times 4 c} = 1$$

$$\frac{a}{n_6} \cdot \frac{D}{c} = \frac{n_4}{n_5} = 7$$

Let $6'' = 1 \text{ unit on } (P_2 \text{ table.})$

$$a = 120 \text{ rev. } n_1 = 120 n_1 \text{ rev./unit}$$

$$\text{Let } n_1 = \frac{1}{4} \text{ so } a = \frac{120}{4} = 30 \text{ rev./unit.}$$

$$\text{from (1)} \quad D = \frac{32 \times 8}{30} =$$

$$\text{from (3)} \quad c = \frac{8 n_4 \left(\frac{32 \times 8}{30} \right)}{32 \times 4} = \frac{8^2}{32 \times 4 \times 2} \cdot \frac{32 \times 8^2}{30} = \frac{8}{30}$$

$$\text{Let } n_4 = \frac{1}{2}$$

$$k = .05 \text{ unit} \\ k_e = .05 \times 60 \times 32 \\ = 120$$

$$\text{from (2)} \quad e = \frac{32 \times 8^2}{8} \cdot 30 = 30 \times 32 \text{ rev./unit.}$$

240

3

$$\text{Let } e = \frac{30 \times 32}{2} \checkmark$$

~~$$e = \frac{30 \times 32}{2} =$$~~

~~$$\eta_4 = \frac{32 \times 4 \times e}{8D} = \frac{32 \times 4 \times 30 \times 32}{8 \times 30 \times D}$$~~

~~$$\text{but } D = \frac{32 \times 8}{30}$$~~

~~$$\eta_4 = \frac{32 \times 4 \times 30 \times 32 \times 8}{8 \times 30 \times 32 \times 8} = 16$$~~

showed by 87

$$\text{Let } e = \frac{30 \times 32}{2}$$

$$c = \frac{32 \times 8 \times 2}{30 \times 32} \frac{8}{15}$$

$$\eta_4 = \frac{32 \times 4 \times 8}{8D15} = \frac{32 \times 4 \times 8 \times 30^2}{8 \times 15 \times 32 \times 8} = 1$$

but if $\eta_5 = 1/2$

$$e = 8/15$$

$$\eta_4 = \frac{32 \times 4 \times e}{8 \eta_5 \times D} = \frac{32 \times 4 \times 8 \times 30^2}{8 \times 32 \times 8 \times 15 \times 1/2 \times 8} = 2$$

trial II.

$$\text{Let } \eta_0 = \frac{1}{2}.$$

$$a = 30$$

$$b = 8$$

$$D = \frac{32 \times 8}{a \eta_0} = \frac{32 \times 8 \times 2}{30}$$

$$\text{Let } e = \frac{30 \times 32}{2}$$

$$\text{SO } c = \frac{32 \times 8}{e} = \frac{\cancel{32} \times 8 \times 2}{30 \times \cancel{32}} = \frac{16}{30}$$

$$\eta_4 = \frac{32 \times 4 \times c}{8 \eta_5 D} = \frac{\cancel{32} \times \cancel{4} \times \frac{16}{30}}{8 \times \cancel{32} \times \frac{30}{2}} = \frac{1}{2}$$

$$\text{Let } \eta_5 = 1$$

$$a = 30$$

$$b = 8$$

$$D = \frac{32 \times 8}{15}$$

$$e = 15 \times 32$$

$$c = \frac{16}{30}$$

$$\eta_1 = \boxed{\frac{1}{4}}$$

$$\eta_2 = \frac{1}{4}$$

$$\eta_3 = \frac{1}{4}$$

$$\eta_4 = \frac{1}{2}$$

$$\eta_5 = 1$$

$$\frac{16 \text{ rev/min.}}{30}$$

$$\frac{16 \text{ in/min.}}{2030}$$

$$\frac{12 \times 360 \times 16}{30 \times 32} = \frac{4320}{960} = 4.5$$

Trial II.

#

~~Let $b = 16$ $\frac{4}{3} = \frac{1}{8}$~~

trial III

5.

$$\text{Let } c = \frac{32}{30}$$

and e must equal $\frac{30 \times 32}{2}$

$$b = \frac{ec}{32} = \frac{32}{32} \frac{30 \times 32}{2} \frac{1}{32} = 16$$

and a = 30

$$\textcircled{a} \frac{n_6 D}{32 \textcircled{b}} = 1$$

try $n_4 = 1/2$

$$\frac{ec}{32 \textcircled{b}} = 1$$

$$n_3 < \frac{40}{206} = \frac{2}{16} = \frac{1}{8}$$

$$\textcircled{b} \frac{n_4 n_3 n_5 D}{32 \textcircled{c}} = 1$$

$$\text{Let } n_3 = \frac{1}{8}$$

$$a = 30$$

$$b = 16$$

$$D = \frac{32 \times 32}{30}$$

$$e = \frac{30 \times 32}{2}$$

$$c = 32/30$$

$$n_1 = 1/4$$

$$n_2 = 1/4$$

$$n_3 = 1/8$$

$$n_4 = 1/2$$

$$n_5 = 1$$

$$n_6 = 1/2$$

$$\text{Let } n_5 = 1$$

$$D = 32 \frac{32}{30} \frac{2}{16} = \frac{32 \times 32}{30}$$

$$n_6 = \frac{32b}{Da} = \frac{32 \cdot 16 \cdot 30}{32 \times 32 \cdot 30} = \frac{1}{2}$$

When $k = .05$ $k \times \frac{30 \times 32}{2} n_2$

$$.1 \times \frac{30 \times 32}{2} \frac{1}{4} = 120 \text{ rev}$$

$$.9375 \frac{9.39}{8.45}$$

$$\sqrt{1.12} = 1.058$$

$$a = \frac{120 n_6}{n_6}$$

$$c = \sqrt{\frac{ea}{n_1 n_2}}$$

$$e = \frac{0.9375}{n_5}$$

$$c n_1 n_2 = \sqrt{ea n_1 n_2}$$

$$D = 32 \sqrt{\frac{e}{a n_1 n_2}}$$

omit
IV

	I	II	III	IV	V	VI	VII	VIII
n_1	} 1/32	1/2	1/2	→ 1/4	1/4			
n_2		1/4	1/4	→ 1/4	1/4			
n_5	1/8	1/4	1/2	→ 1/2	1/4			
n_6	1/8	1/4	1/4	→ 1/4	1/4			
a	15	30	30	→ 30	30			
e	7.5	3.75	1.875	→ 1.875 2.085	3.75			
D	128	32	$32/\sqrt{2}$	32	$32\sqrt{2}$			
c		30	$30/\sqrt{2}$	30				
Travel of Drum c/n_2	1.88	3.75	2.66	1.88	2.66			
Slip Scale in/rev			.53	$3/8$ *.375 0.3995				

time of solution

Travel of Drum

Slip Scale in/rev

← 2

April 3, 1931

$$20 \times \frac{9.39}{180} = 1.043 \text{ rev/unit}$$

$$e = \frac{1.043}{1/4} = 2.085 \text{ rev/unit}$$

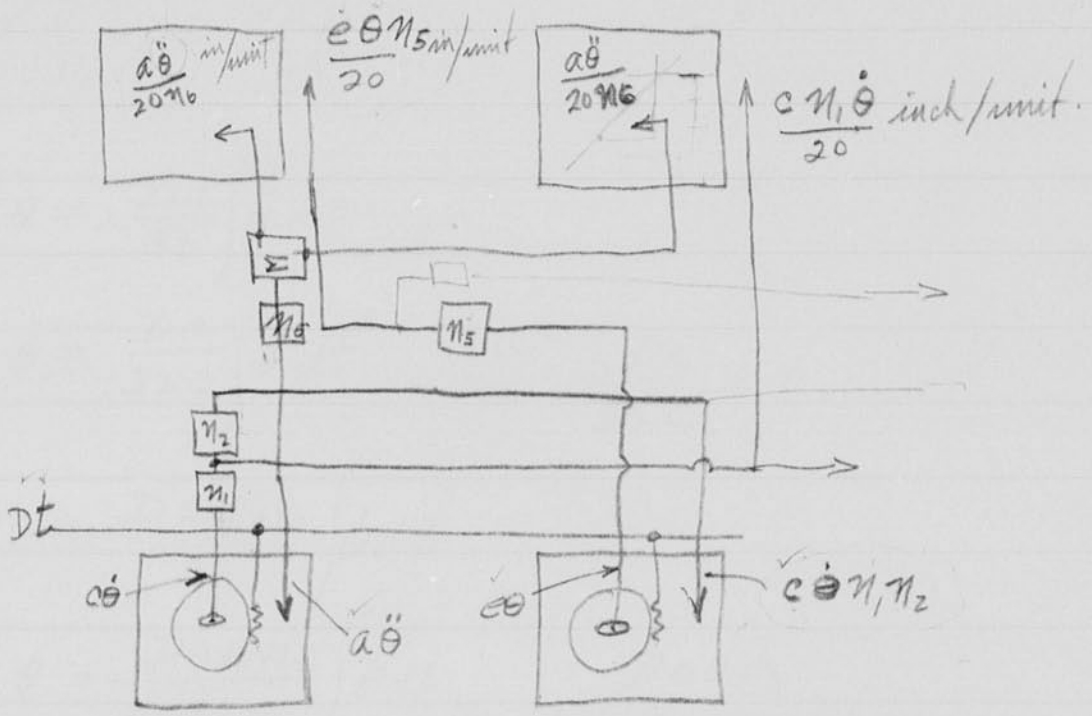
$$c = \sqrt{\frac{ea}{n_1 n_2}} = \sqrt{16 \cdot 2.085 \cdot 30} = 4\sqrt{62.5}$$

$$= 4 \cdot 7.90 = 31.6 \text{ rev/unit}$$

$$c \times \frac{1}{4} = 31.6 \cdot \frac{1}{4} = 7.9 \text{ rev/unit}$$

$$\frac{7.9}{20} = 0.395 \text{ in./unit}$$

$\frac{a}{n_6} = k e^{-n_5}$



$a\ddot{\theta} \leq 40 \text{ rev.}$
 $c\dot{\theta} n_1, n_2 \leq 40 \text{ rev.}$
 $\frac{c n_1, \dot{\theta}}{20} \leq 12'' \pm$

$\dot{\theta} = \frac{Da}{32e} \int \ddot{\theta} dt$
 $\theta = \frac{c n_1, n_2 D}{32e} \int \dot{\theta} dt$
 $\theta = \frac{c n_1, n_2 D^2 a}{32^2 c e} \int \ddot{\theta} dt$

Conditions for scales is that $\frac{c n_1, n_2 D^2 a}{32^2 c e} = 1.$

Speed of solutions

$\ddot{\theta} = \frac{c n_1, n_2 D^2 a}{32^2 c e} \ddot{\theta} = k$
 $\frac{a\ddot{\theta}}{20 n_6} = k \frac{n_5 e \theta}{a}$
 $\ddot{\theta} = \frac{n_6 k n_5 e}{a} \theta$
 $\ddot{\theta} = \frac{k n_5 c n_6}{a} \theta$

At one point

(1)

$$\frac{a \ddot{\theta}}{20 n_6} = k \frac{e \theta n_5}{20}$$

$$c \dot{\theta} = \frac{D a}{32} \dot{\theta} dt$$

$$\dot{\theta} = \frac{D a}{32 c} \dot{\theta} dt \quad \therefore \frac{D a}{32 c} = 1$$

$D = \frac{32c}{a}$

$$e \theta = -\frac{D c n_1 n_2}{32} \dot{\theta} dt$$

$$\theta = -\frac{D c n_1 n_2}{32 e} \dot{\theta} dt \quad \therefore \frac{D c n_1 n_2}{32 e} = 1$$

$$\theta = -\frac{D^2 a c n_1 n_2}{32^2 c e} \int \dot{\theta} dt^2$$

$$\ddot{\theta} = k \frac{e n_5 n_6}{a} \theta \quad k \text{ is geometrical slope}$$

$$\theta = -k \frac{D^2 a c n_1 n_2 \cdot e n_5 n_6}{32^2 c e a} \int \theta dt^2$$

$$\ddot{\theta} = -k \frac{D^2 n_1 n_2 n_5 n_6}{32^2} \theta$$

$$\text{If } \theta = A \sin(\omega t + \alpha)$$

$$\omega = \sqrt{k \frac{D^2 n_1 n_2 n_5 n_6}{32^2}}$$

$$T_{\text{sec}} \text{ for 1 period } = \frac{120 \pi D}{N}$$

$$T = \frac{2\pi}{\omega} \quad \text{in terms of } t$$

T_D in terms of variable (DL) is D times T

②

$$T_{\text{time in seconds}} = \frac{\text{sec.}}{\text{rev. of time sh.}} \times \frac{\text{rev.}}{\text{unit of } DT} \times \frac{\text{units of } t}{\text{unit of } \frac{DT}{t}}$$

In example set up for backlash test

$$k=1 \quad n_6=1 \quad n_5=\frac{1}{4}$$

$$\frac{a}{n_6} = k \cdot n_5 \quad a = \frac{e}{4}$$

$$n_1 n_2 = \frac{1}{4}$$

$$c = \frac{Da}{32}$$

$$c = \frac{32 \cdot 4}{D} = \frac{32 \cdot 16a}{D}$$

$$\frac{Da}{32} = \frac{32 \cdot 16a}{D}$$

$$D^2 = 32 \cdot 16$$

$$D = 4 \times 32 = 128$$

$$c = \frac{128a}{32} = 4a = e$$

Time of oscillation in sec. = 88

Speed of time shaft = N rpm $\frac{\text{sec}}{\text{rev}} = \frac{60}{N}$

$$88 = \frac{60}{N} 128 \times 2\pi$$

$$N = \frac{60 \times 128 \times 2\pi}{88} = \frac{160 \times 32 \pi}{11} = \frac{1920\pi}{11} = 550 \text{ rpm}$$

$$\begin{aligned} \text{Motor speed} &= 3.75 \times 550 \\ &= 2070 \text{ rpm} \end{aligned}$$

Edgerton's problem (New ratios (II))

$$\frac{e n_5}{20} = \frac{0.9375}{20} = 0.046875 \text{ in/deg.}$$

$$\checkmark e = 4 \times 0.9375 = 3.75 \text{ rev/deg @ output of \#2 integ.}$$

$$\frac{a}{20 n_6} = 6 \text{ inches per unit torque}$$

$$n_6 = \frac{1}{4} \quad \checkmark a = \frac{20 \times 6}{4} = 30 \text{ rev of \#1 lead sc. / unit torque}$$

$$\frac{D c n_1 n_2}{32 e} = 1 \quad n_1 n_2 = \frac{1}{8}$$

$$D = \frac{32 e}{n_1 n_2 c} = \frac{32 \times 3.75 \times 8}{c} = \frac{256 \times 3.75}{c}$$

$$= \frac{960}{c}$$

$$D = \frac{32 c}{a} = \frac{32}{30} c \quad c = \frac{30}{32} D$$

$$D = \frac{960}{\frac{30}{32} D} \quad D^2 = \frac{32 \times 256 \times 3.75}{30}$$

$$D^2 = \frac{2^{13} \times 3.75}{30} = \frac{2^{12} \times 3.75}{15} = 2^{12} \times 0.25$$

$$= 2^{10}$$

$$\checkmark D = 32$$

$$\checkmark c = 30$$

Try in $c \& n_1 n_2 \leq 40$

$$e = \frac{30}{8} \dot{\theta} = \frac{30 \times 16}{8} = 60$$

Edgerton's problem

3

Take $e = 0.9375 \times 4 = 3.75$ rev./degree of θ on input
Corresponds to input table \angle scale 0.1

Just to Handen Table

New layout of ratios (II)

(4)

$$\omega^2 = k \frac{D^2 n_1 n_2 n_5 n_6}{32^2} = \frac{32^2}{32^2 \times 8 \times 4 \times 4} = \frac{1}{128} = \frac{1}{8^2 \cdot 2}$$

$$T = 2\pi 8\sqrt{2} = 16\sqrt{2}\pi = 71 \text{ units of } t$$

$$= 71 \times 32 \text{ revolutions}$$

$$= 2270$$

$$T = \frac{120\pi}{N} 32$$

$$\frac{2270}{400} = 5.67 \text{ min.}$$

Old ratios (I) $n_6 = \frac{1}{8}$ $n_5 = \frac{1}{8}$

$$n_1, n_2 = \frac{1}{32}$$

$$\omega^2 = \frac{128^2}{32 \times 8 \times 8 \cdot 32^2} = \frac{2^{14}}{2^{15+6}} = 2^{-7} = \frac{1}{128}$$

$$\omega = \frac{1}{8\sqrt{2}}$$

$$T = 2\pi 8\sqrt{2} = 16\sqrt{2}\pi \text{ units of } t$$

$$= 71 \times 128 \text{ revolutions of } t \text{ shaft}$$

$$\text{Time in min.} = \frac{71 \times 128}{400} = \frac{71 \times 32}{100} = 22.7 \text{ min.}$$

$$T = \frac{120\pi}{N} 128$$

Just to Handson Table

Scales III c ($c_{III} = \frac{c_{II}}{\sqrt{2}}$)

(5)

$$\frac{Da}{32c} = 1$$

$$\frac{32 \times 30}{32 \times 30} = 1$$

$$\frac{Dc n_1 n_2}{32e} = 1$$

$$e n_5 = 0.9375$$

$$e = \frac{0.9375}{n_5}$$

$$\frac{Dc n_1 n_2 n_5}{32 \times 0.9375} = 1$$

✓ Check on new values

$$\frac{32 \times 30}{32 \times 0.9375 \times 8 \times 4} = 1$$

$$D = \frac{32c}{a}$$

$$\frac{32c}{a} \cdot \frac{c n_1 n_2 n_5}{32 \times 0.9375} = 1$$

$$n_1 = \frac{1}{2}$$

$$n_2 = \frac{1}{4}$$

Take $c = \frac{30}{\sqrt{2}}$ find n_5

$$\frac{32}{30} \frac{30^2}{2} \frac{1}{8} \frac{n_5}{32 \times 0.9375} = 1$$

$$n_5 = \frac{30 \times 2 \times 8 \times 32 \times 0.9375}{32 \times 30^2} = \frac{1}{2}$$

$$\omega^2 = k \frac{D^2 n_1 n_2 n_5 n_6}{32^2} = k \frac{32^2 c^2 \cdot n_1 n_2 n_5 n_6}{32^2 a^2}$$

$$D = \frac{32 \cdot 30}{30 \cdot \sqrt{2}} = \frac{32}{\sqrt{2}}$$

$$T = \frac{120 \pi}{N} \frac{32}{\sqrt{2}}$$

$$\frac{22.5 \cdot 30 \sqrt{2}}{32 \cdot 30} = 1 \quad \checkmark$$

$$\frac{22.5 \cdot 30}{32 \sqrt{2}} \frac{1}{8} \frac{1}{1.875} = \frac{675}{670} = 1.0 \quad \checkmark$$

$c n_1 n_2 \dot{\theta} < 40 \text{ rev.}$

$$\frac{30}{\sqrt{2}} \frac{1}{8} 17$$

$$\frac{32.30^2}{32 \times 8 \times 4 \times 0.9375} \quad \star /$$

Scales IV

Take $n_1, n_2 = \frac{1}{16}$ $c = 30\sqrt{2}$

$$\frac{32c}{a} \cdot \frac{c n_1 n_2 n_5}{32 \times 0.9375} = 1$$

$$\frac{32 \cdot 30^2 \cdot 2}{30 \times 32 \times 0.9375 \times 16} n_5 = 1$$

$$n_5 = \frac{30 \times 32 \times 76 \times 0.9375}{32 \times 30^2 \times 2} = \frac{75}{30} = \frac{5}{4}$$

$$\omega^2 = k \frac{c^2 n_1 n_2 n_5 n_6}{a^2}$$

$$D = \frac{32c}{a} = \frac{32 \cdot 30\sqrt{2}}{30} = 32\sqrt{2}$$

$$T_{sec} = \frac{60}{N} D \cdot 2\pi = \frac{120\pi D}{N}$$

$$= \frac{120\pi}{N} \cdot 32\sqrt{2}$$

$c n_1 n_2 \cdot 6$

$$\frac{30\sqrt{2}}{1} \cdot \frac{1}{16} \cdot 17 = 45$$

$$\frac{d\theta}{dt} dt.$$

$$\theta = \int \left[\int P_2 dt - \int k(1 - b \cos 2\theta) \dot{\theta} dt \right] dt.$$

→ θ
→ $\dot{\theta}$
→ $\int P_2 dt$
→ $k(1 - b \cos 2\theta)$
→ $\int k(1 - b \cos 2\theta) \dot{\theta} dt$
→ $\int \left[\int P_2 dt - \int k(1 - b \cos 2\theta) \dot{\theta} dt \right] dt$
→ θ

Just to Hayden Feb

Apr 5
1.

$\dot{\theta}$ max is about 2 or less.

so $a < \frac{40}{2} = 20$ \therefore let $a = \underline{16}$

Max for e units = 0.2

$e \times 0.2 < 40$ $e < \frac{40}{0.2} = 200$. Let $e = \underline{128}$

$\dot{\theta}$ max is about 20

$b\dot{\theta} < 40$ $b < \frac{40}{20} = 2$. let $b = \underline{2}$.

1. $1 = \frac{\eta_3 \eta_6 a D}{32 b} = \frac{\eta_3 \eta_6 16 D}{32 \times 2} = \frac{\eta_3 \eta_6 D}{4}$

2. $1 = \frac{\eta_7 e c}{b 32} = \frac{\eta_7 128 c}{2 32} = 2 \eta_7 c$

3. $1 = \frac{\eta_4 \eta_5 b D}{32 c} = \frac{\eta_4 \eta_5 2 D}{32 c} = \frac{\eta_4 \eta_5 D}{16 c}$

η_3 ✓
 η_6 ✓
 η_7 ✓
 η_4 ✓
 η_5 ✓
 D
 c

I Try $\eta_5 = 1$ $\eta_6 = 1$ $\eta_3 = 1/4$ $\eta_7 = 1/4$

1. $1 = \frac{\eta_3 \eta_6 D}{4} = \frac{1}{4} \frac{1 D}{4}$ $D = 16$.

$\eta_5 = \frac{16 c}{D \eta_4}$
 $= \frac{16 \times 4}{64} = 1$

2. $1 = 2 \eta_7 c = 2 \frac{1}{4} c$ $c = 2$.

3. $1 = \frac{\eta_4 \eta_5 D}{16 c} = \frac{\eta_4 1 D}{16 \times 2}$ $\eta_4 = 2$. but $\eta_4 \leq \frac{1}{2}$.

II try $\eta_4 = 1/2$ which gives from 3 $\frac{D}{c} = \frac{16 \times 2}{\eta_5}$

Let $\eta_3 = \eta_7 = 1/4$

$\eta_6 = 1/4$

$D = \frac{4}{\frac{1}{4} \frac{1}{4}} = 16 \times 4 = 64$

$c = \frac{1}{2 \times \frac{1}{4}} = 2$.

$\eta_5 = \frac{c}{D} \frac{16 \times 2}{1} = \frac{2}{64} \times 32 = 1$ 2.

D
 c
 η_5 3
 η_6
 η_7
 η_3

✓

III try $n_4 = 1/2$ (same as II)

Let $n_3 = n_7 = 1/2$

$n_6 = 1/2$

$$c = \frac{1}{2 \cdot \frac{1}{2}} = 1$$

$$D = \frac{4}{\frac{1}{2} \cdot \frac{1}{2}} = 16$$

$$n_5 = \frac{32}{D} c = \frac{32}{16} = 2$$

NS

IV try $n_4 = 1/4$

$$\frac{n_3 n_6 D}{4} = 1$$

try $n_3 = n_7 = 1/4$

$$2 n_7 c = 1$$

$n_6 = 1$

$$\frac{1 n_5 D}{4 \times 16} = 1$$

$$D = \frac{4}{\frac{1}{4} \cdot 1} = 16 \quad c = \frac{1}{2 \cdot \frac{1}{4}} = 2$$

$$n_5 = \frac{64 D}{D} = \frac{64 \cdot 2}{16} = 8 \quad \text{NS}$$

Check on case II

- n_1
- n_2
- n_3 14
- n_4 14
- n_5 1
- n_6 114
- n_7 14
- a 16
- b 2
- c 2
- e 128
- D 64

$$1. \quad 1 = \frac{n_3 n_6 a D}{32 b} = \frac{14 \cdot 14 \cdot 16 \cdot 64}{32 \times 2} = 1 \quad \checkmark$$

$$2. \quad 1 = \frac{n_7 c c}{b 32} = \frac{14 \cdot 128 \cdot 2}{2 \times 32} = 1 \quad \checkmark$$

$$3. \quad 1 = \frac{n_4 n_5 b D}{32 c} = \frac{14 \cdot 1 \cdot 2 \cdot 64}{32 \times 2} = 2 \quad \times$$

Joint to Hayden Feb

$$\frac{400}{20} = \frac{20 \text{ in}}{\text{min}}$$

$$1.043$$

3

V
3

$$c = 1.043 \text{ rev/unit.}$$
~~$$c = 1.043$$~~
~~$$c = 1.043$$~~

$$n_4 = \frac{1}{2} \quad 1.056$$

$$\frac{c}{n_4} = c n_2 = \underline{2.086 \text{ rev/unit.}}$$

$$c = 1.043 \text{ rev/unit}$$

Quantities determined

$$c = 1.043 \text{ rev/unit.}$$

$$a = 16 \text{ rev}$$

$$e = 128 \text{ rev}$$

$$b = 2 \text{ rev.}$$

$$1. \quad i = \frac{n_3 n_6 a D}{32 b} = \frac{n_3 n_6 16 D}{32 \cdot 2} = \frac{n_3 n_6 D}{4}$$

$$2 \quad A = \frac{n_2 e c}{632} = \frac{n_2 128 1.043}{632} = \frac{4 n_2 1.043}{2}$$

$$n_7 = \frac{2}{4 \cdot 1.043} = \frac{1}{2 \cdot 1.043}$$

Replot chart so that $c = 1.0$.

Let $n_8 = 1$ $c = 1.0 \text{ rev per unit. (scaled deg.)}$

$$\text{or } \frac{1.0 \times 18^\circ}{2^\circ} = 9 \text{ inches for } 180 \text{ feet } \underline{\text{degrees.}}$$

Just to Hayden Feb

VI

Quantities determined

$c = 1.0$

$a = 16$

$e = 128$

$b = 2$

1. $1 = \frac{n_3 n_6 a D}{32 b} = \frac{n_3 n_6 \times D}{32 \times 2} = \frac{n_3 n_6 D}{4}$

2. $1 = \frac{n_7 e c}{b^3 2} = \frac{n_7 \times 128 \times 1}{2 \times 32} = 2 n_7$

$n_7 = \frac{1}{2}$

3. $1 = \frac{n_4 n_5 b D}{32 c} = \frac{n_4 n_5 \times D}{32 \times 1} = \frac{n_4 n_5 D}{16}$

(5 unknowns) n_3, n_4, n_5, n_6, D (two eqns)

Let $n_3 = 1/2, n_4 = 1/2, n_5 = 1$

from 3 $1 = \frac{1/2 \times 1}{16} D \quad D = 16 \times 2 = 32$

from 1 $1 = \frac{1/2 \times n_6 \times 32^2}{4} = 4 n_6$

Retabulation

- $n_1 \quad 1/8$
- $n_2 \quad 1/4$
- $n_3 \quad 1/2$
- $n_4 \quad 1/2$
- $n_5 \quad 1$
- $n_6 \quad 1/4$
- $n_7 \quad 1/2$
- $a \quad 16$
- $b \quad 2$
- $c \quad 1$
- $D \quad 32$
- $e \quad 128$

$n_6 = 1/4$

for $\frac{a^6}{20 n_1} = \frac{1.6^6}{20} = 6.4$ in per unit

$\frac{e}{n_2} = \frac{128}{20 \times 1/8} = \frac{128 \times 8}{20} = 5.12$ in per unit

VII For cases where $k = 0.05$ as max.

Then $e = 256$.

a remains 16

b 2

c 1

Unit equations $1 = \frac{\eta_3 \eta_6 a D}{32 b} = \frac{\eta_3 \eta_6 \times 6 D}{32 \times 2} = \frac{\eta_3 \eta_6 D}{4}$

$\eta_7 = 1/4$

$1 = \frac{\eta_7 e c}{632} = \frac{\eta_7 \times 256 \times 1}{2 \times 32} = \eta_7 \times 4$

$1 = \frac{\eta_4 \eta_5 b D}{52 c} = \frac{\eta_4 \eta_5 \times 2 D}{32 \times 1} = \frac{\eta_5 \eta_4 D}{16}$

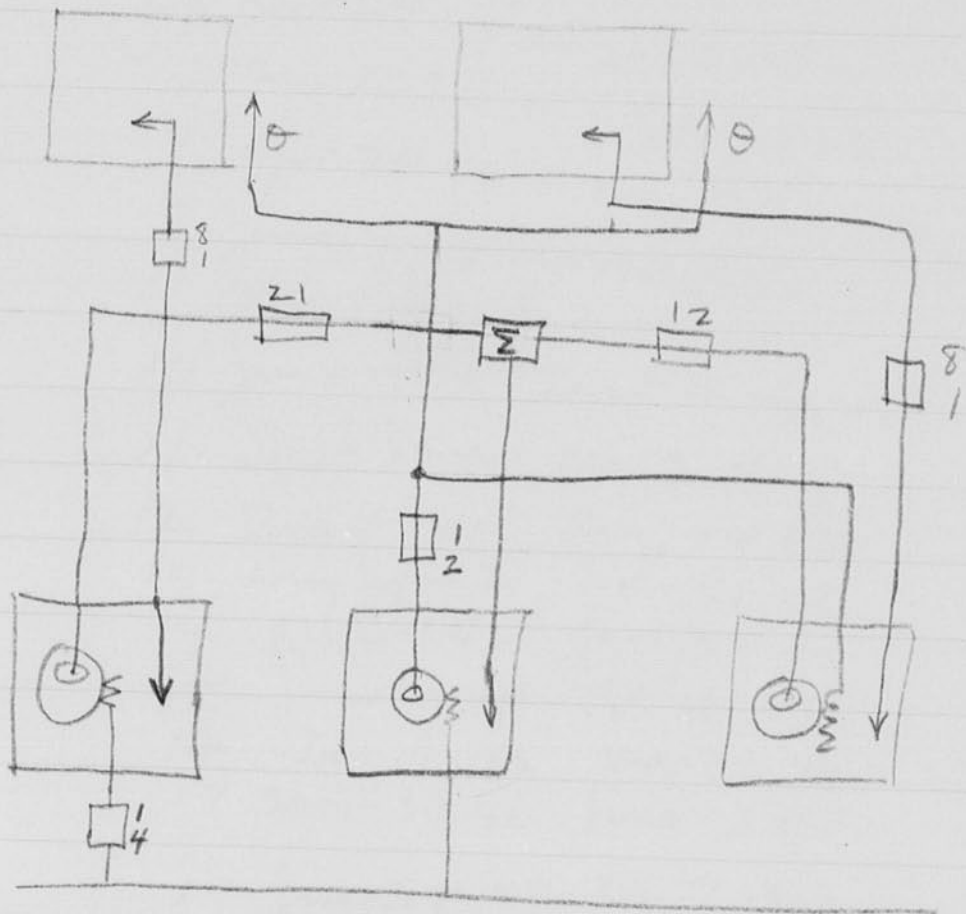
~~$\frac{ds}{dt} + \frac{R}{P_j} (1 - b \cos 2\theta) s = \frac{P_L}{P_j}$~~

~~$s = \frac{do}{dt}$~~

$\frac{d^2 s}{dt^2} + \left(\frac{P_d}{P_j} \right) (1 - b \cos 2\theta) \frac{ds}{dt} = \left(\frac{P_L}{P_j} \right)$

$A \frac{d^2 g}{dt^2} + \frac{R}{L} \frac{dg}{dt} = \frac{E}{L}$

Print to Handson Feb



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Computation

Apr 21 1951

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T-II July 20 31 Jan 12 32

T-5 Oct 27 34 Aug 27 35

T-6 Aug 27 35 - Apr 25 36

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10 June 13 39 Sept 17 40

11 Sept 17 40 Dec 3 41

12 Dec 4 41 Aug 24 42

13 Aug 24 42 Mar 31 43

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EGG Dec 8 48 April 8, 1951

Underwater U.P. Aug 4 52 Oct 19 1952

Photography 20 Feb 7 50 Dec 27 51

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22 Jan 9 54 Apr 19 55

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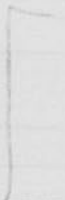
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H. Elgerton Jan 21 1967 (see white cover book)

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HAROLD E. EDDERTON
52 MASS. AVE.
CAMBRIDGE MASS.

M.I.T. ROOM 4-210

FEB. 15, 1930.

SELF-HUNTING
OF
SYNCHRONOUS MACHINES.

P102-105 Stubs.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

COMPUTATION BOOK

GENERAL INSTRUCTIONS

In all work in which *accuracy* and *ease of reference* are important, much depends upon carrying out the computation in a systematic manner. The following instructions, taken from the *Engineering Department Figuring Book of the Allis-Chalmers Co.*, serve as a guide in this matter.

"All computations, of whatever kind, are to be made in these books, except in cases where special blanks may be provided for specific kinds of computation. Computations may be made in ink or pencil, whichever may be more convenient. Pencil figuring should be done with a soft pencil. All the work of computation should be done in these books, including all detail figuring."

"Each subject should begin on a new page, no matter how much space may be left on the previous page. The subject, with the date of beginning it, should be plainly written at the top of the first page of the subject."

"Work should be done systematically, and as neatly as consistent with rapidity. The books are, however, intended for convenience, and no unnecessary work should be done for sake of appearance only. Errors should be crossed off instead of erased, except where the latter will facilitate the work. Work should not be crowded. Paper costs less than the time which would be expended in attempting to economize space in making erasures."

"Where curves drawn on section paper (or sketches) are necessary parts of a computation, they should be pasted in the book, except where specifically otherwise provided for."

"Computations should be indexed, in the back of the book, by the person using the book."

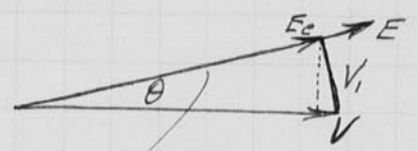
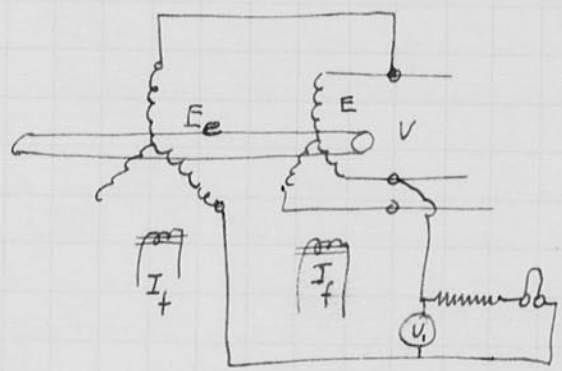
* * * * *

Harold E. Edgerton

Feb. 15 1930
H. S. Edgerton

Oscillographic Measurement of the Angular Displacement in a Syn. Machine.

An extra synchronous machine on the shaft of a machine being tested gives a voltage which ~~is in~~ has a definite relationship with the induced emf. E of that machine ~~being tested since~~ since the field poles have a rigid relationship. The open-circuit voltage from this extra machine is combined vectorially with the terminal voltage of the machine under test and a trigonometric 3 voltage problem gives the angular displacement.

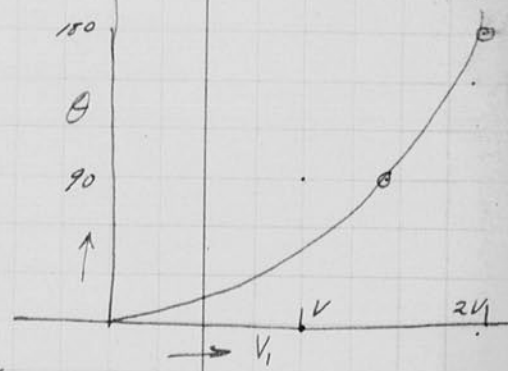


field poles of both machines in line.

If E_e and V are ^{not} equal then $\theta = \cos^{-1} \frac{V^2 + E_e^2 - V_1^2}{2 E_e V}$

When E_e and V are equal then

$$\theta = \cos^{-1} \frac{2V^2 - V_1^2}{2V^2} = \cos^{-1} \left(1 - \frac{V_1^2}{2V^2} \right)$$



This method has been used in the past with great success.

When the rotor is slipping, for instance during a transient following a sudden load, then the length of E_e depends directly upon the speed and some small errors are introduced. The expression for angle is then

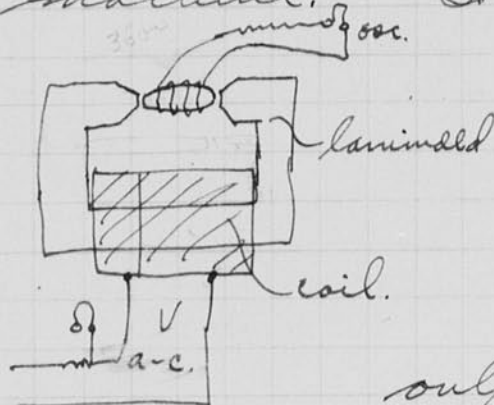
$$\theta = \cos^{-1} \frac{V^2 + (1-s)^2 E_e^2 - V_1^2}{2 E_e V (1-s)}$$

where s is the slip. The slip is usually less than 2% so this is ordinarily neglected.

Angle Measurement.

Feb 15 1930
H. Edgerton

I had Mr. Vershaw build me a device which I hoped would aid in measuring angle with the oscillograph. This was to be a variable reluctance motor, its stator supplied by a.c. from the terminals, and its rotor circuit to be connected to an oscillograph vibrator. A sketch is shown of the machine. It was driven at 3600 r.p.m.



It was driven at 3600 r.p.m. by a 2:1 gear on a 1800 r.p.m. act in the dynamo laboratory (Siemens wave generator set 99 a. b. c.).

Needless to say the output voltage of the rotor was of a very peculiar shape. The current input was also interesting and offers some possibility of future development. The current to the exciting a.c. coil has a nick in it every time that the eccentric rotor lines up with the pole faces. If the speed is constant this will occur at the same place on the current wave. Otherwise it will occur at different places and its position will depend upon the relative positions of the rotor and the terminal voltage.

Trouble was experienced with the copper brushes that were used and finally caused the experiment to be discontinued after four osc. were taken.

Data for osc.

- Osc. 1. Chattering brushes on rotor gave poor record. Film also too slow.
 Osc. 2. Poor Focus. Wave interference. Too slow drum speed.
 Osc. 3. Ok. on page 5.
 Osc. 4. Focus poor.
 Angle voltage V , as explained on page 3 is shown on all osc.

Notebook # 3

Filming and Separation Record

1 unmounted photograph(s)

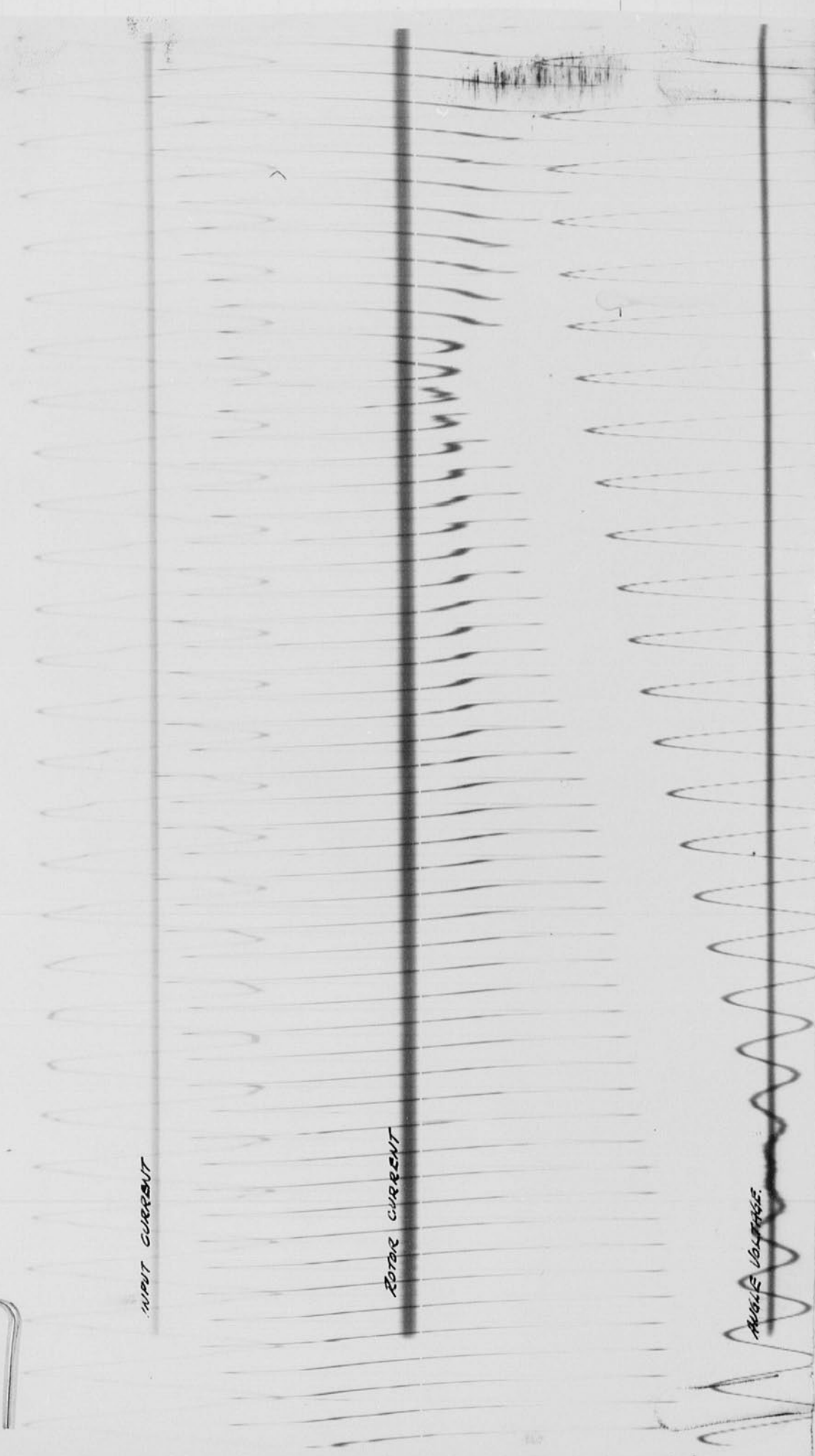
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Osc. 3
FEB. 15 1930 H.E.E.
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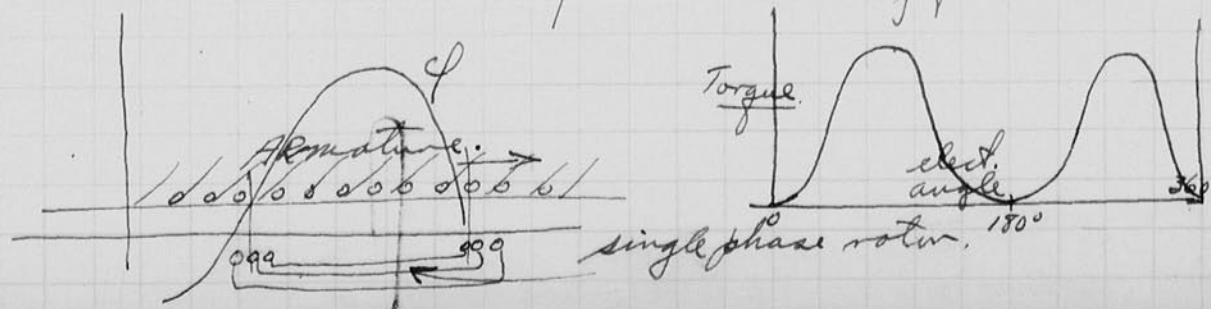
Source of Energy for Self Oscillations
of Synchronous Machines.

7
Feb. 21, 1930
S. E. Edgerton.

A recent paper by Nickle and Pierce in the A. I. E. E. blames self hunting on armature resistance. The unbalance of the rotor circuit is also considered and it is noted that pole face damping windings in the quadrature axis suppresses the possibility of self-oscillation. No physical picture was given in this paper of the mechanism whereby ~~self~~ a synchronous machine should hunt accumulatively. Our common reasoning powers tell that if a winding on the rotor is oscillated back and forth during an oscillation, the induced currents from Len's law always oppose any change of position. Such an action would tend to reduce oscillations.

I have been thinking about this problem for several years, trying to satisfy myself that it was possible from the characteristics of synchronous and induction machines to explain self-hunting or self sustained oscillations.

My present opinion is that conditions for negative damping are only possible under conditions of unbalance on the rotor circuit. A rotor that is single phase has a torque that pulsates at double slip frequency. It is a maximum when the coil is cutting the maximum flux of the rotating field. It is also a minimum when not cutting the rotating field.



As shown in the sketch on the preceding page this torque due to a single phase winding is a minimum at zero angular displacement. If the currents in the single phase rotor effect the rotating field, ϕ , then this field can be represented by two oppositely rotating fields. This is necessary since the flux of the rotating field is pulsating.

The positively rotating field reacts with the field winding and gives positive damping according to some law as the $\sin^2 \theta$ as a function of angle and directly proportional to the slip for small values.

The negative ^{field} however is in the opposite sense always to the positive and due to ~~the~~ circuit requirements does not have its maximum at the same time the positive field lines up with the coil. This being true the negative field at small angles is in such a direction as to give torque which appears to push instead of pull as a function of slip. This causes oscillations to build up. They will continue to increase in magnitude until large enough angles are reached in the course of the swinging where the positive damping again occurs.

Mechanical-Electrical Torque Equation.

9
July 23 1930
L. E. Edgerton.

The swinging of a synchronous machine is always in accord with the differential equation that states the sum of the various mechanical and electrical torques that exist during the disturbance. This equation is of the form:

$$F_s \frac{d^2\theta}{dt^2} + f(\theta) \cdot F \left(\frac{d\theta}{dt} \right) + f'(\theta) = \text{load torque.}$$

In an ideal machine with negligible armature resistance, a smooth rotor, and for small values of slip, this reduces to approximately the following:

$$F_s \frac{d^2\theta}{dt^2} + F_d \frac{d\theta}{dt} + F_m \sin\theta = \text{load torque.}$$

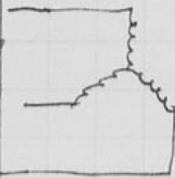
The term $F_d \frac{d\theta}{dt}$ is such that it always opposes any force that tries to change the speed of the machine from synchronism. This effect then in a machine with a balanced rotor circuit always tends to damp out oscillations. Such damping is termed by definition as being positive.

In order to get self-oscillations this force must have a negative sense some of the time since there can be no source of oscillation in the synchronizing or inertia terms of the differential equation. Dreyfus points out this fact. A rotor with a single phase winding will give this condition for small angular displacements.

Induction Motor with a Single-phase Rotor.

Feb 23, 1930
J. E. Edgerton

Balanced
3 ϕ voltage
supply
V volts/ph.



From the open phase of the rotor to the short-circuited pair there will be a voltage of slip frequency. This may be split up into its positive and negative sequence components by Fortescue's method of symmetrical phase components.

Since the current in the open circuited phase is zero the sum of the positive and negative sequence currents are zero in this phase and thus equal to each other in magnitude but 180° in phase

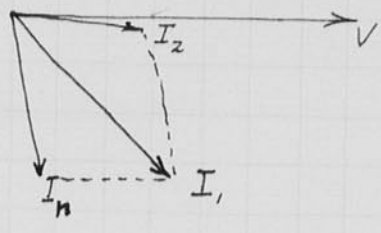
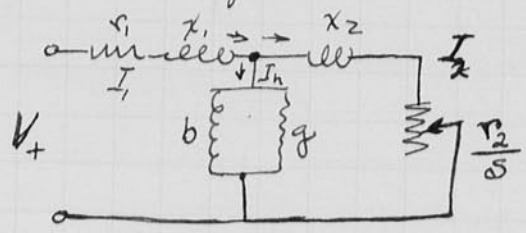
$$I = I_{2+} + I_{2-} = 0 \quad \text{where the 2 refers to the rotor circuit.}$$

I think that this problem can be best analysed by considering that the stator has two components of voltage applied to it. Each will be polyphase and balanced. One will be the impressed e.m.f., V . The other will depend upon the slip at which the rotor is considered. It will be of such a value that the rotor current in one phase is zero. Such a requirement states that this fictitious voltage shall have a freq. of $(1-s)f$ where s is the slip.

The stator current will be the sum of the two current components and since they are at different frequencies the total stator current will contain beats which occur in the different phases in rotation. I know this to be the case since I have observed it in the laboratory, both with meters and with the oscillograph.

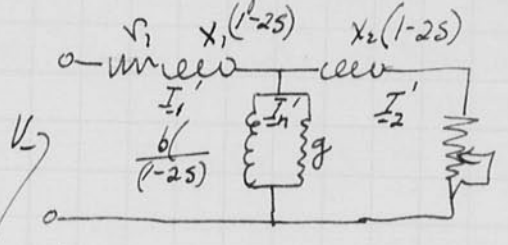
Feb 23 1930
H. E. Edgerton

Equivalent circuit for impressed voltage V_+



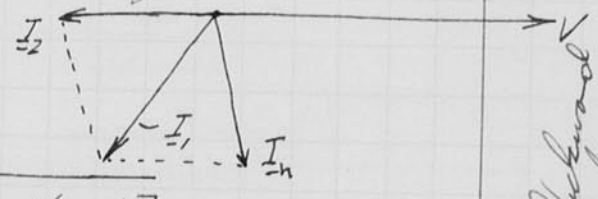
$$I_2 = \frac{(V_+ - I_n Z_2)}{r_1 + \frac{r_2}{s} + j(x_2 + x_1)}$$

Equivalent circuit for $(1-2s)$ voltage



Generator action for slips from 0 to .5. Motor then on down to slip of 1. or stand still.

frequency of $(1-2s)$ times that of V_+



$$I_2' = \frac{V_+ - I_n Z_2}{r_1 - \frac{r_2 s}{1-2s} + j[x_2(1-2s) + x_1(1-2s)]}$$

These two currents in the rotor winding are to be equal and opposite in one phase that is open.

$I_2 = I_2'$ from the above equations we can find V_- in terms of V_+ and the slip.

$$V_- - I_n Z_2 = \frac{Z_T}{Z_T} (V_+ - I_n Z_1) = (V_+ - I_n Z_1) \frac{Z_T}{(r_1 + \frac{r_2 s}{1-2s}) + j(1-2s)(x_1 + x_2)}$$

$$V_- = (V_+ - I_n Z_1) \frac{Z_T}{Z_T} + I_n Z_2$$

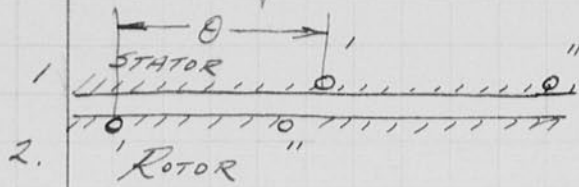
$$Z_+ = r_1 + jx_1$$

$$Z_- = r_1 + jx_1(1-2s)$$

This scheme is not so good. The voltage for the backward flux component should be just in the rotor. Feb. 24, 1930.

Differential equation of a two phase machine and the reduction of these equations to a simple form.

Feb 24/1930
L. E. Edgerton



Sinusoidal flux is assumed in this treatment. The air gap is also uniform and the iron has a constant permeability.

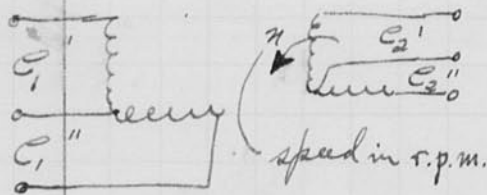
Voltage drops around phase one of the stator.

$$e_1' = r_1 i_1' + L_1 p i_1' + M p i_2' \cos \theta + M p i_2'' \cos \theta + \frac{\pi}{2} \quad (1)$$

$$\text{phase 2. } e_1'' = r_1 i_1'' + L_1 p i_1'' + M p i_2' \cos \theta + M p i_2'' \cos \theta - \frac{\pi}{2} \quad (2)$$

$$\text{Rotor ph. 1. } e_2' = r_2 i_2' + L_2 p i_2' + M p i_1' \cos \theta + M p i_1'' \cos \theta - \frac{\pi}{2} \quad (3)$$

$$\text{ph 2. } e_2'' = r_2 i_2'' + L_2 p i_2'' + M p i_1' \cos \theta + M p i_1'' \cos \theta + \frac{\pi}{2} \quad (4)$$



These may be reduced from four equations and four unknowns to two equations, two unknowns by splitting the currents into components.

In order to do this let:

$$\begin{aligned} i_1' &= i_{41} + i_1 & i_2' &= i_{42} + i_2 & e_1' &= e_{41} + e_1 \\ \text{and } i_1'' &= -j i_{41} + j i_1 & i_2'' &= -j i_{42} + j i_2 & e_1'' &= -j e_{41} + j e_1 \end{aligned}$$

This change of variables gives the differential equations a new form wherein there are only ~~two~~ two variables and two equations $e_2' = e_{42} + e_2$ $e_2'' = -j e_{42} + j e_2$.

$$e_{41} = (r_1 + L_1 p) i_{41}' + M p i_{42}' \epsilon^{i\theta}$$

$$e_1 = (r_1 + L_1 p) i_1' + M p i_2' \epsilon^{-i\theta}$$

$$e_{42} = (r_2 + L_2 p) i_{42}' + M p i_{41}' \epsilon^{-i\theta}$$

$$e_2 = (r_2 + L_2 p) i_2' + M p i_1' \epsilon^{i\theta}$$

Notice the change of sign for the stator and rotor equations is only in the exponent.

Notebook # 3

Filming and Separation Record

___ unmounted photograph(s)

___ negative strip(s)

13 unmounted page(s) *12 page reprint with one page*
(notes, drawings, letters, etc.) *of notes inserted*

was/were filmed where originally located between page 12 and 13.

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EDGERTON

Preliminary proof. Subject to revision.

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Two-Reaction Theory of Synchronous Machines

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Synopsis.—Starting with the basic assumption of no saturation, in addition, new and more accurate equivalent circuits are derived for synchronous machines equipped with field-pole salient poles of any arbitrary construction. Special detailed formulas are also developed for the determination of current and torque on the rotor during starting, and when only small deviations from an average operating angle are involved. It is proposed to continue the analysis in a subsequent paper.

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THIS paper presents a generalization and extension of the work of Blondel, Dreyfus, and Doherty and Nicke, and establishes new and general methods of calculating current power and torque in salient and non-salient pole synchronous machines, under both transient and steady load conditions.

Attention is restricted to symmetrical three-phase machines with field structure symmetrical about the axes of the field winding and interpolar space, but salient poles and an arbitrary number of rotor circuits are considered.

Idealization is resorted to, to the extent that saturation and hysteresis in every magnetic circuit and eddy

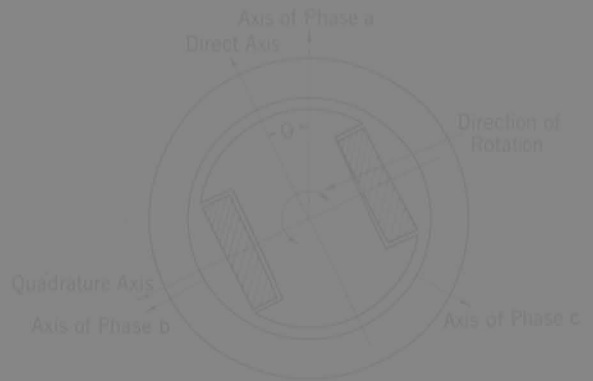


FIG. 1

currents in the armature iron are neglected, and in the assumption that, as far as concerns effects depending on the position of the rotor, each armature winding may be regarded as, in effect, sinusoidally distributed.³

A. Fundamental Circuit Equations

Consider the ideal synchronous machine of Fig. 1, and let

¹General Eng. Dept., General Electric Company, Schenectady, N. Y.
²Single-phase machines may be regarded as three-phase machines with one phase open circuited.
³Stator for a machine with stationary field structure.
⁴For numbered references see Bibliography.
 Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

i_a, i_b, i_c = per unit instantaneous phase currents
 e_a, e_b, e_c = per unit instantaneous phase voltages
 ψ_a, ψ_b, ψ_c = per unit instantaneous phase linkages
 t = time in electrical radians

$$p = \frac{d}{dt}$$

Then there is

$$\begin{aligned} e_a &= p\psi_a - r i_a \\ e_b &= p\psi_b - r i_b \\ e_c &= p\psi_c - r i_c \end{aligned} \quad (1)$$

It has been shown previously⁴ that

$$\begin{aligned} \psi_a &= I_a \cos \theta - I_b \sin \theta \\ &- \frac{x_0}{3} (i_a + i_b + i_c) - \frac{x_d + x_q}{3} \left[i_a - \frac{i_b + i_c}{2} \right] \\ &- \frac{x_d - x_q}{3} (i_a \cos 2\theta + i_b \cos (2\theta - 120) \\ &\quad + i_c \cos (2\theta + 120)) \end{aligned}$$

$$\begin{aligned} \psi_b &= I_a \cos (\theta - 120) - I_b \sin (\theta - 120) \\ &- \frac{x_0}{3} \frac{i_a + i_b + i_c}{3} - \frac{x_d + x_q}{3} \left[i_b - \frac{i_c + i_a}{2} \right] \\ &- \frac{x_d - x_q}{3} (i_b \cos (2\theta - 120) + i_c \cos (2\theta + 120) \\ &\quad + i_a \cos 2\theta) \end{aligned} \quad (2)$$

$$\begin{aligned} \psi_c &= I_a \cos (\theta + 120) - I_b \sin (\theta + 120) \\ &- \frac{x_0}{3} \frac{i_a + i_b + i_c}{3} \\ &- \frac{x_d + x_q}{3} \left[i_c - \frac{i_a + i_b}{2} \right] \\ &- \frac{x_d - x_q}{3} (i_c \cos (2\theta + 120) + i_a \cos 2\theta \\ &\quad + i_b \cos (2\theta - 120)) \end{aligned}$$

where,

Page 2
7-5 52

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On the other hand, if n additional rotor circuits exist in the direct axis there is,

$$E = I + X_{11d} I_{1d} + X_{21d} I_{2d} + \dots + X_{ndd} I_{ndd} + (x_d - x_d') i_d$$

where,

I_{1d}, I_{2d}, \dots , etc., are the per-unit instantaneous currents in circuits 1, 2, etc., of the direct axis, X_{11d}, X_{21d}, \dots , etc., are per-unit mutual coefficients between the field and circuits 1, 2, etc., of the direct axis.

Similar relations exist for the linkages in each of the additional rotor circuits except $x_d - x_d'$ is to be replaced by a term x_{nd} . However, since all of these additional circuits are closed, it follows that there is an operational result

$$I_d = I + I_{1d} + I_{2d} + \dots + I_{ndd} = G(p) E + H(p) i_d \tag{4}$$

where E is the per-unit value of the instantaneous field voltage, and $G(p)$ and $H(p)$ are operators such that

$$\begin{aligned} G(0) &= 1 & G(\infty) &= 0 \\ H(0) &= 0 & H(\infty) &= x_d - x_d' \end{aligned}$$

$x_d' =$ the subtransient reactance*

It will be convenient to write $H(p) = x_d - x_d(p)$ and to rewrite (4) in the form,

$$I_d = G(p) E + [x_d - x_d(p)] i_d \tag{4a}$$

If there are no additional rotor circuits, there is, as shown in Appendix 1,

$$\begin{aligned} \Psi &= I - (x_d - x_d') i_d \\ E &= T_d p \Psi + I \end{aligned}$$

where T_d is the open circuit time constant of the field in radians.

There is then,

$$\begin{aligned} G(p) &= \frac{1}{T_d p + 1} \\ x_d(p) &= \frac{x_d' T_d p + x_d}{T_d p + 1} \end{aligned}$$

*This definition is somewhat different from that given in reference 2.

it in the direct axis there is,

$$E = I + [X_{11q} I_{1q} + X_{21q} I_{2q} + \dots + X_{nqq} I_{nqq} + (x_q - x_q') i_q]$$

where,

$$I_q = \frac{2}{3} [i_q \cos \theta + i_q \cos(\theta - 120) + i_q \cos(\theta + 120)]$$

which gives,

$$i_q(p) = x_q - \frac{[X_{11q} T_d p + X_{21q} T_d p + \dots + X_{nqq} T_d p] p + 1}{A(p)}$$

$$A(p) = [X_{11q} - X_{11d}] T_d T_{d1} p^2 + [X_{11q} T_d + T_{d1}] p + 1$$

If there is more than one additional rotor circuit the operators $G(p)$ and $x_d(p)$ will be more complicated but may be found in the same way.

The effects of external field resistance may be found by changing the term I in the field voltage equation to $R I$. Open circuited field corresponds to R equal to infinity.

Similarly, there will be

$$I_q = [x_q - x_q(p)] i_q \tag{5}$$

where,

$$i_q = -\frac{2}{3} [i_q \sin \theta + i_q \sin(\theta - 120) + i_q \sin(\theta + 120)] \tag{3a}$$

$$x_q(0) = x_q, x_q(\infty) = x_q'$$

So far, 10 equations have been established relating the 15 quantities $e_d, e_q, e_0, i_{1d}, i_{2d}, i_{ndd}, i_{1q}, i_{2q}, i_{nq}, \psi_{1d}, \psi_{2d}, \psi_{ndd}, \psi_{1q}, \psi_{2q}, \psi_{nq}, I_d, I_q, E, \theta$ in a general way.

It follows that when any five of the quantities are known the remaining 10 may be determined. Their determination is very much facilitated, however, by the introduction of certain auxiliary quantities $e_d, e_q, e_0, i_{1d}, \psi_{1d}, \psi_{1q}, \psi_{10}$.

Thus, let

$$e_0 = \frac{1}{3} [e_d + e_q + e_0] \tag{3b}$$

$$e_d = \frac{2}{3} [e_d \cos \theta + e_q \cos(\theta - 120) + e_0 \cos(\theta + 120)]$$

$$e_q = -\frac{2}{3} [e_d \sin \theta + e_q \sin(\theta - 120) + e_0 \sin(\theta + 120)] \tag{6}$$

$$e_0 = \frac{1}{3} [e_d + e_q + e_0]$$

$$\psi_{1d} = \frac{2}{3} [\psi_{1d} \cos \theta + \psi_{2d} \cos(\theta - 120) + \psi_{10} \cos(\theta + 120)]$$

$$\psi_{1q} = -\frac{2}{3} [\psi_{1d} \sin \theta + \psi_{2d} \sin(\theta - 120) + \psi_{10} \sin(\theta + 120)] \tag{7}$$

$$\psi_{10} = \frac{2}{3} [\psi_{1d} \sin \theta + \psi_{2d} \sin(\theta - 120) + \psi_{10} \sin(\theta + 120)]$$

$$\psi_{2d} = -\frac{2}{3} [\psi_{1d} \sin \theta + \psi_{2d} \sin(\theta - 120) + \psi_{10} \sin(\theta + 120)]$$

$$\psi_{2q} = \frac{2}{3} [\psi_{1d} \sin \theta + \psi_{2d} \sin(\theta - 120) + \psi_{10} \sin(\theta + 120)]$$

$$\psi_{20} = -\frac{2}{3} [\psi_{1d} \sin \theta + \psi_{2d} \sin(\theta - 120) + \psi_{10} \sin(\theta + 120)]$$

$$\psi_{ndd} = \frac{2}{3} [\psi_{1d} \cos \theta + \psi_{2d} \cos(\theta - 120) + \psi_{10} \cos(\theta + 120)]$$

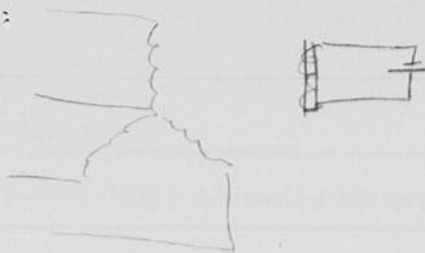
$$\psi_{ndq} = -\frac{2}{3} [\psi_{1d} \sin \theta + \psi_{2d} \sin(\theta - 120) + \psi_{10} \sin(\theta + 120)]$$

$$\psi_{nd0} = \frac{2}{3} [\psi_{1d} \sin \theta + \psi_{2d} \sin(\theta - 120) + \psi_{10} \sin(\theta + 120)]$$

G(p) = 1 / (T_d p + 1)

Nov. 26, 1930

i_a
 i_b
 i_c



Round Rotor
no excitation in quadrature axis.
no ~~field~~ ~~current~~ ~~current~~
 $x_d = x_q$

$$\psi_a = I_d \cos \theta - I_q \sin \theta - \frac{x_d}{3} (i_a + i_b + i_c) - \frac{x_d + x_q}{3} \left[i_a - \frac{i_b + i_c}{2} \right]$$

$$\psi_a = I_d \cos \theta - I_q \sin \theta - \frac{2x_d}{3} \left[i_a - \frac{i_b + i_c}{2} \right]$$

$$\psi_b = I_d \cos(\theta - 120) - I_q \sin(\theta - 120) - \frac{2x_d}{3} \left[i_b - \frac{i_c + i_a}{2} \right]$$

$$\psi_c = I_d \cos(\theta + 120) - I_q \sin(\theta + 120) - \frac{2x_d}{3} \left[i_c - \frac{i_a + i_b}{2} \right]$$

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$$e_a = \frac{1}{3} [\cos \theta p \psi_a + \cos(\theta - 120) p \psi_a + \cos(\theta + 120) p \psi_a] p \theta$$

$$= \frac{2}{3} [\cos \theta p \psi_a + \cos(\theta - 120) p \psi_a] p \theta$$

$$= \frac{2}{3} [\cos \theta p \psi_a + \cos(\theta + 120) p \psi_a]$$

$$= \frac{2}{3} [\cos \theta p \psi_a + \cos(\theta - 120) p \psi_a + \cos(\theta + 120) p \psi_a] p \theta$$

$$= e_a + r i_a + \psi_a p \theta$$

$$p \psi_a = -\frac{2}{3} [\sin \theta p \psi_a + \sin(\theta - 120) p \psi_a + \sin(\theta + 120) p \psi_a] p \theta$$

$$= -\frac{2}{3} [\sin \theta p \psi_a + \sin(\theta - 120) p \psi_a] p \theta$$

$$= -\frac{2}{3} [\sin \theta p \psi_a + \cos(\theta + 120) p \psi_a] p \theta$$

$$= -\frac{2}{3} [\sin \theta p \psi_a + \cos(\theta - 120) p \psi_a + \cos(\theta + 120) p \psi_a] p \theta$$

$$= -\frac{2}{3} [\sin \theta p \psi_a + \cos(\theta - 120) p \psi_a + \cos(\theta + 120) p \psi_a] p \theta$$

$$= -\frac{2}{3} [\sin \theta p \psi_a + \cos(\theta - 120) p \psi_a + \cos(\theta + 120) p \psi_a] p \theta$$



Fig. 2

angles θ , $\theta - 120$ and $\theta + 120$, where taking the direct axis as the axis of reals,

$$\begin{aligned} \bar{v} &= v_a + j v_b \\ \bar{\psi} &= \psi_a + j \psi_b \\ \bar{i} &= i_a + j i_b \end{aligned}$$

If we introduce in addition the vector quantity,

$$\bar{I} = I_a + j I_b$$

the circuit equations previously obtained may be



Fig. 3

transferred into the corresponding vector forms,

$$\begin{aligned} \bar{v} &= p \bar{\psi} - \bar{r} + [p \theta] j \bar{\psi} \\ \bar{\psi} &= \bar{I} - \bar{z} \end{aligned}$$

where, $\bar{z} = x_a i_a + j x_b i_b$

Fig. 3 shows these relations graphically.

B. Armature Power Output

The per-unit instantaneous power output from the armature is necessarily proportional to the sum

Also it may be readily verified that

$$v_a = I_a - x_a i_a = G(p) E - x_a(p) i_a \quad (11)$$

$$v_b = I_b - x_b i_b = -x_b(p) i_b \quad (12)$$

$$v_c = -x_c i_c \quad (13)$$

Equations (8) to (13) establish six relatively simple relations between the 11 quantities $v_a, v_b, v_c, i_a, i_b, i_c, \psi_a, \psi_b, \psi_c, E, \theta$. In practice it is usually possible to determine five of these quantities directly from the terminal conditions, after which the remaining six may be calculated with relative simplicity. After the direct, quadrature, and zero quantities are known the phase quantities may be determined from the identical relations

$$\begin{aligned} i_a &= i_c \cos \theta - i_b \sin \theta + i_0 \\ i_b &= i_c \cos(\theta - 120) - i_a \sin(\theta - 120) + i_0 \quad (14) \\ i_c &= i_a \cos(\theta + 120) - i_b \sin(\theta + 120) + i_0 \\ \psi_a &= \psi_c \cos \theta - \psi_b \sin \theta + \psi_0 \\ \psi_b &= \psi_c \cos(\theta - 120) - \psi_a \sin(\theta - 120) + \psi_0 \quad (15) \\ \psi_c &= \psi_a \cos(\theta + 120) - \psi_b \sin(\theta + 120) + \psi_0 \\ e_a &= e_c \cos \theta - e_b \sin \theta + e_0 \end{aligned}$$

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during normal operation at unity power factor it may be seen that the factor of proportionality must be 2/3.

Suppose that the constant slip of the rotor is s . Then there is,

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Substituting from Equations (14) and (16) there results the useful relation,

$$\psi_d = -x_q(p) i_q$$

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Putting $p x_d(p) + r = z_d(p)$

C. Electrical Torque on Rotor

on the rotor directly from the general relation
 [Total power output] =
 [mechanical power transferred across gap] + [rate of decrease of total stored magnetic energy] - [total ohmic losses]

$$e_d = p G(p) E - z_d(p) i_d + (1-s) x_d(p) i_q \quad (20)$$

$$E - x_d(p) i_d - z_q(p) i_q \quad (21)$$

Solving gives,

$$i_d = \{ [p z_q(p) + (1-s)^2 x_d(p)] G(p) E - z_q(p) e_d + (1-s) r G(p) e_q \} \div D(p) \quad (22)$$

$$i_q = \frac{(1-s) r G(p) E - z_d(p) e_q + (1-s) x_d(p) e_d}{D(p)} \quad (23)$$

where, $D(p) = z_d(p) z_q(p) + (1-s)^2 x_d(p) x_q(p)$

E. Two Machines Connected Together

Suppose that two machines which we will designate respectively by the subscripts g and h , are connected together, but not to any other machines or circuits, and assume in addition that there are no zero quantities. In this case the voltages of each machine will be equal

However, since this torque depends uniquely only on the magnitudes of the currents in every circuit of the machine, it follows that a general formula for torque may be derived by considering any special case in which arbitrary conditions are imposed as to the way in which these currents are changing as the rotor moves.

The simplest conditions to impose are that I_a, I_e, i_a, i_e and i_e remain constant as the rotor moves. In this case there will be no change in the stored magnetic energy of the machine as the rotor moves, and the power output of the rotor will be just equal in magnitude and opposite in sign to the rotor losses. It follows that under the special conditions assumed, Equation (18) becomes simply,

[armature power output] =
 [mechanical power across gap] - [armature losses]

$$\text{or, } P = T p \theta - \frac{2r}{3} \{ i_a^2 + i_b^2 + i_c^2 \}$$

$$= T p \theta - r \{ i_d^2 + i_q^2 + i_o^2 \}$$

Then,

T = per-unit instantaneous electrical torque

$$= \frac{e_d i_d + e_q i_q + e_o i_o + r \{ i_d^2 + i_q^2 + i_o^2 \}}{p \theta}$$

but subject to the conditions imposed,

$$e_d = -\psi_d p \theta - r i_d$$

$$e_q = \psi_d p \theta - r i_q$$

$$e_o = -r i_o$$

It therefore follows that,

$$T = i_q \psi_d - i_d \psi_q \quad (19)$$

$$= \text{vector product of } \vec{\psi} \text{ and } \vec{i} \quad (19a)$$

$$= \vec{\psi} \times \vec{i}$$

a result which could have been established directly by physical reasoning. Formula (19) is employed by Dreyfus in his treatment of self-excited oscillations of synchronous machines.¹⁴

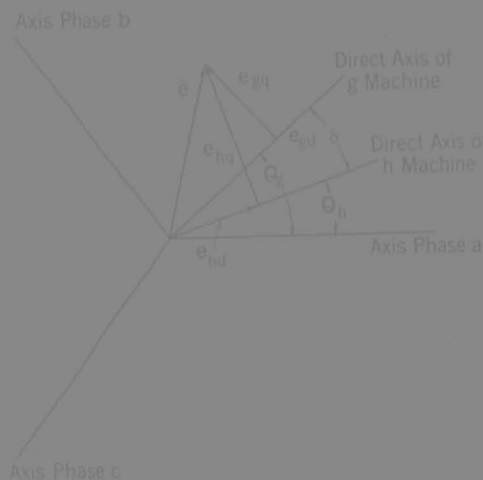


FIG. 4

phase for phase, and it therefore follows that the voltage vectors of each machine must coincide, as shown in Fig. 4.

Referring to the figure it will be seen that the direct and quadrature components of voltage of the two machines are subject to the mutual relations,

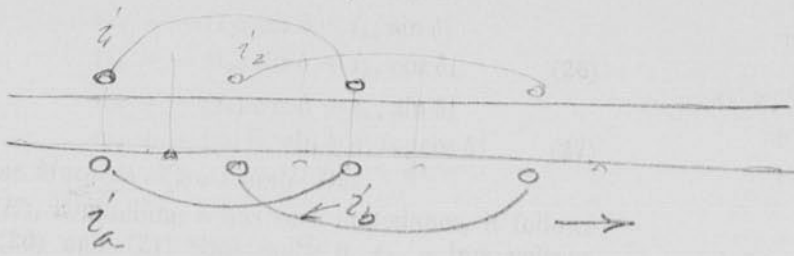
$$e_{hd} = e_{gd} \cos \delta - e_{gq} \sin \delta$$

$$e_{hq} = e_{gd} \sin \delta + e_{gq} \cos \delta \quad (24)$$

$$e_{gd} = e_{hd} \cos \delta + e_{hq} \sin \delta$$

$$e_{gq} = -e_{hd} \sin \delta + e_{hq} \cos \delta \quad (25)$$

On the other hand, the induced voltage will be



$$\omega = \frac{d\theta}{dt}$$

$$\theta = \int \omega dt$$

$$e_1 = (r_1 + Lp)i_1 + Mp(i_2' \cos \omega t) + Mp(i_1' \sin \omega t)$$

$$e_2 = (r_2 + Lp)i_2 + Mp(i_1' \cos \omega t) + Mp(i_2' \sin \omega t)$$

$$e_a = (r_a + L_a p)i_a + Mp(i_1' \cos \omega t) + Mp(i_2' \sin \omega t)$$

$$e_b = (r_b + L_b p)i_b + Mp(i_1' \cos \omega t) + Mp(i_2' \sin \omega t)$$

A derivation of this formula for steady load conditions has been previously given by Dehery and Nickle.

III. Three-Phase Star Circuit with Constant Rotor Speed

Since a three-phase short circuit causes ω_r and ω_s to vary rapidly, the effect with respect to rotor speed maintained may be found by assuming $\omega_r = \omega_s = \omega$ in (22) and (23) where ω_r and ω_s are the values of ω_r and ω_s before the short circuit. The initial currents existing before the short circuit must be added to the currents found in this way in order to obtain the resultant current after the short circuit.

With $\omega_r = \omega_s = \omega$, and thus $\omega = \omega_s$ in detail

$$i_1(p) = \frac{E_1(p) + \dots}{D(p)}$$

$$i_2(p) = \frac{E_2(p) + \dots}{D(p)}$$

The working out of the formulas may be illustrated by consideration of the simple case of a machine with no rotor circuits in addition to the field. In this case there is

$$i_1(p) = \dots$$

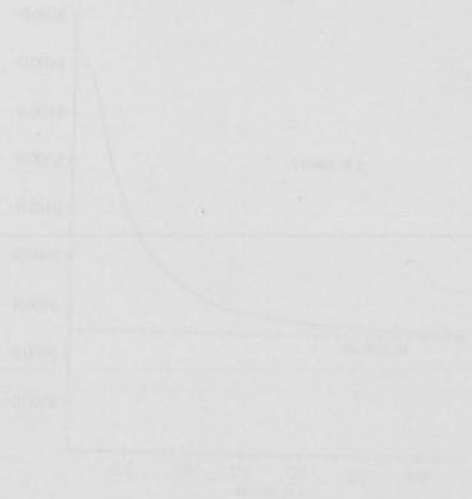
$$i_2(p) = \dots$$

$$D(p) = \dots$$

where the summation is extended over the roots of

$$p^2 + 2\sigma p + \omega^2 = 0$$

The phase currents may, of course, be found from



Equation (22) by the coefficients of Equation (14)

For the particular case

the roots \dots of the equation \dots were found to be \dots

It will be noted that, as would necessarily be the

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On the other

$$i_{ad} = -i_{aa} \cos \delta - i_{aa} \sin \delta$$

$$i_{aq} = -i_{aa} \sin \delta + i_{aa} \cos \delta$$

F. One Machine on an Infinite Bus

In (E), if machine *b* has zero impedance, it follows

From (20) and

Then for

$$e_a = e_{a0}$$

$$e_b = e_{b0}$$

G. Torque Angle Relations

From Equations (11), (12), and (19), there is

$$T = \frac{I_a \psi_a}{x_a} - \frac{I_d \psi_d}{x_d} - \frac{I_q \psi_q}{x_q}$$

Then if the rotor leads the vector $\vec{\psi}$ by an angle δ there is

$$\psi_a = -\psi \sin \delta$$

$$\psi_d = \psi \cos \delta$$

$$T = \frac{I_a \psi}{x_a} \cos \delta + \frac{I_d \psi \sin \delta}{x_d} + \frac{x_d - x_q}{2 x_d x_q} \psi^2 \sin 2 \delta \quad (29)$$

A derivation of this formula for steady load conditions has been previously given by Doherty and Nickle.⁴

H. Three-Phase Short Circuit with Constant Rotor Speed Maintained

Since a three-phase short circuit causes e_d and e_q to vanish suddenly, its effect with constant rotor speed maintained may be found by impressing $e_d = -e_{d0}$, $e_q = -e_{q0}$ in (22) and (23) where e_{d0} and e_{q0} are the values of e_d and e_q before the short circuit. The initial currents existing before the short circuit must be added to the currents found in this way in order to obtain the resultant current after the short circuit.

With $s = 0$ and E constant there is in detail

$$i_d = \frac{x_d(p) e_{d0} + x_q(p) e_{q0}}{D(p)} \cdot 1 + \frac{x_d E - r e_{d0} - x_d e_{d0}}{r^2 + x_d x_q}$$

$$i_q = \frac{x_d(p) e_{q0} - x_q(p) e_{d0}}{D(p)} \cdot 1 + \frac{r E - r e_{q0} + x_d e_{q0}}{r^2 + x_d x_q} \quad (30)$$

The working out of the formulas may be illustrated by consideration of the simple case of a machine with no rotor circuits in addition to the field. In this case there is

$$x_q(p) = x_q$$

$$x_d(p) = \frac{x_d' T_0 p + x_d}{T_0 p + 1}$$

$$D(p) = \left\langle \frac{x_d' T_0 p + x_d}{T_0 p + 1} p + r \right\rangle (x_d p + r)$$

$$+ \frac{x_d' T_0 p + x_d}{T_0 p + 1} x_q$$

$$+ r (e_{d0} + x_d + r T_0) + x_d' x_d T_0 p$$

$$+ r^2 + x_d x_q$$

$$T_0 p + 1$$

(31)

By the expansion theorem there is, finally,

$$i_d = \frac{x_d E}{r^2 + x_d x_q} + \sum_1 \frac{(T_0 \alpha_n + 1) ((x_d \alpha_n + r) e_{d0} + x_d e_{d0}) \epsilon^{-\alpha_n t}}{\alpha_n d'(\alpha_n)}$$

$$i_q = \frac{r E}{r^2 + x_d x_q} + \sum_1 \frac{(x_d' T_0 \alpha_n^2 + (x_d + r T_0) \alpha_n + r) e_{q0} - (T_0 \alpha_n x_d' + x_d) e_{d0}}{\alpha_n d'(\alpha_n)} \epsilon^{-\alpha_n t} \quad (32)$$

where the summation is extended over the roots of

$$d(\alpha) = 0 \text{ and } d'(p) = \frac{d}{d p} d(p)$$

The phase currents may, of course, be found from

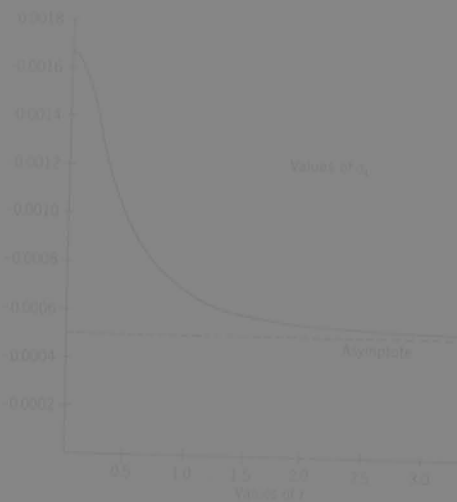


FIG. 5

Equations (32) by the application of Equations (14). For the particular case

$$T_0 = 2,000, x_d = 1.00, x_q = 0.60, x_d' = 0.30$$

the roots $\alpha_1, \alpha_2, \alpha_3$ of the equation $d(p) = 0$, were found to be as shown in Figs. 5, 6, and 7, where

$$\alpha_2 = \alpha_1 + \alpha_3$$

$$\alpha_3 = \alpha_1 - \alpha_2$$

It will be noted that, as would necessarily be the

case, where $r =$
short circuit time
 $r = 0,$

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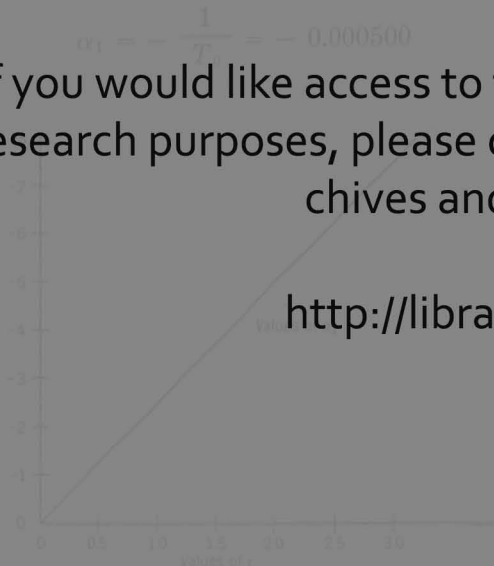


FIG. 6

The root α_1 is found to be almost exactly equal to the value which it would have were $T_s = \infty, i. e.,$

$$\alpha_1 = \frac{r(x_d' + x_q)}{2x_d'x_q} \text{ approximately}$$

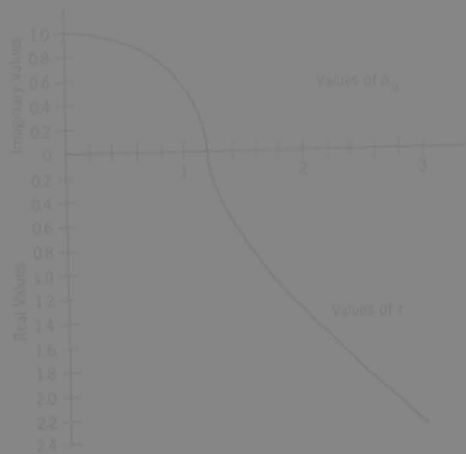


FIG. 7

Thus, in the special case considered this approximate formula gives

$$\alpha_1 = \frac{(0.30 + 0.60)r}{2 \times 0.30 \times 0.60} = 2.50r$$

which checks the result found by the exact solution of the cubic.

I. Starting Torque

On infinite bus and with slip s , there will be, choosing

$$e_d = \cos \delta T$$

$$ed = 1.0$$

$$e_q = -j$$

$$r [j s x_d' (j s) + r] + (1-s)^2 x_d' (j s) x_d' (j s) + j (1-2s) x_d' (j s) - r$$

$$= \left\{ j x_d' (j s) - \frac{r}{1-2s} \right\} \div \{ x_d' (j s) x_d' (j s) + \frac{r}{1-2s} + j s (x_d' (j s) + x_q' (j s)) \} \quad (34)$$

$$i_s = -\frac{[j s x_d' (j s) + r] (-j) - (1-s) x_d' (j s)}{r^2 + (1-2s) x_d' (j s) x_d' (j s) + j s r [x_d' (j s) + x_q' (j s)]}$$

$$= \left\{ x_d' (j s) + \frac{j r}{1-2s} \right\} \div \{ x_d' (j s) x_d' (j s) + \frac{r}{1-2s} [r + j s (x_d' (j s) + x_q' (j s))] \} \quad (35)$$

The expressions for average power and torque then become,

$$P_{av} = 1/2 [e_d \cdot i_d + e_q \cdot i_q]$$

$$T_{av} = 1/2 [i_q \cdot \psi_d - i_d \cdot \psi_q]$$

where the dot indicates the scalar product, or

$$P_{av} = 1/2 [1 \cdot i_d - j \cdot i_q]$$

$$= 1/2 [\text{Real of } i_d - \text{Imaginary of } i_q] \quad (36)$$

There is in general,

$$e_d + r i_d = p \psi_d - (1-s) \psi_q$$

$$e_q + r i_q = (1-s) \psi_d + p \psi_q$$

$$\psi_d = \frac{\begin{vmatrix} e_d + r i_d - (1-s) \psi_q \\ e_q + r i_q - p \end{vmatrix}}{\begin{vmatrix} p & -(1-s) \\ (1-s) & p \end{vmatrix}}$$

$$= \frac{p(e_d + r i_d) + (1-s)(e_q + r i_q)}{p^2 + (1-s)^2} \quad (37)$$

$$\psi_q = \frac{p(e_q + r i_q) - (1-s)(e_d + r i_d)}{p^2 + (1-s)^2} \quad (38)$$

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$$\psi_s = \frac{js(e_s + r i_s) + (1-s)(e_s + r i_s)}{1-2s}$$

$$\psi_s = \frac{js(-j + r i_s) - (1-s) - r(1-s) i_s}{1-2s}$$

$$= -j + \frac{r}{1-2s} (js i_s + (1-s) i_s) \quad (39)$$

$$\psi_s = \frac{js(-j + r i_s) - (1-s) - r(1-s) i_s}{1-2s}$$

$$= \frac{- (1-2s) + r[js i_s - (1-s) i_s]}{1-2s}$$

$$= -1 + \frac{r}{1-2s} [js i_s - (1-s) i_s] \quad (40)$$

Thus,

$$T_{av} = 1/2 \begin{bmatrix} i_s (-j) + i_s \cdot \frac{r}{1-2s} (js i_s + (1-s) i_s) \\ -i_s (-1) - i_s \cdot \frac{r}{1-2s} (js i_s - (1-s) i_s) \end{bmatrix}$$

$$= P_{av} + \frac{r}{2(1-2s)} \begin{bmatrix} (1-s)(i_s^2 + i_d^2) \\ + 2s i_s \cdot j i_d \end{bmatrix}$$

$$= P_{av} + \frac{r}{2} (i_s^2 + i_d^2) + \frac{rs}{2(1-2s)} \begin{bmatrix} i_s^2 + i_d^2 \\ + 2 i_s \cdot j i_d \end{bmatrix}$$

$$= P_{av} + r \frac{i_s^2 + i_d^2}{2} + \frac{rs}{2(1-2s)} (i_s + j i_d)^2 \quad (41)$$

Mr. Ralph Hammar, who has been engaged in the application of the general method of calculation outlined above, to the predetermination of the starting torque of practical synchronous motors, has suggested an interesting modification of formulas (36) and (41), based upon the fact that, since the total m. m. f. consists of direct and quadrature components pulsating at slip frequency, it may be resolved into two components, one moving forward at a per-unit speed $1-s+s=1.0$, and the other moving backward at a per-unit speed $1-s-s=1-2s$. Thus from this standpoint half of both the direct and quadrature components will move forward, and half backward. Since the quadrature axis is ahead of the direct it follows that as far as concerns the forward component the quadrature current i_s is equivalent to a d-c. $j i_s$, while as regards backward component it is equivalent to a direct component

It follows that the vector amounts of forward and backward m. m. f. or current are

$$\text{forward current} = i_s + j i_d$$

$$\text{backward current} = i_s = \frac{1}{\sqrt{2}} (i_s - j i_d) \quad (42)$$

and the vector amounts of forward and backward voltage are

$$\text{forward voltage} = \frac{r}{2} (e_s + j e_d)$$

$$\text{backward voltage} = \frac{r}{2} (e_s - j e_d) \quad (43)$$

There is,

$$i_s = \frac{1}{2} \left\{ \frac{-2r}{1-2s} + j[x_d'(js) + x_q'(js)] \right\} + \left\{ x_d'(js)x_q'(js) + \frac{r}{1-2s}(r + js[x_d'(js) + x_q'(js)]) \right\}$$

$$i_s = \frac{1}{2} \left\{ j[x_d'(js) - x_q'(js)] \right\} + \left\{ x_d'(js)x_q'(js) + \frac{r}{1-2s}(r + js[x_d'(js) + x_q'(js)]) \right\}$$

$$e_s = 1.0 \quad (44)$$

$$e_d = 0 \quad (45)$$

$$P_{av} = e_s \cdot i_s = \text{real of } i_s \quad (46)$$

$$T_{av} = P_{av} + r i_s^2 + \frac{r}{1-2s} i_d^2 \quad (47)$$

J. Zero Armature Resistance, One Machine Connected to an Infinite Bus

Assume that a machine of negligible armature resistance is operating from an infinite bus of per-unit voltage e , at synchronous speed, with a steady excitation voltage E_s , and displacement angle δ_s . At the instant $t = 0$, let δ and E change.

There is,

$$i_d = \frac{E_s - \psi_{d0}}{x_d} - \frac{1}{x_d(p)} \Delta \psi_d + \frac{G(p)}{x_d(p)} \Delta E$$

$$i_q = -\frac{\psi_{q0}}{x_q} - \frac{1}{x_q(p)} \Delta \psi_q$$

$$\psi_d = e \cos \delta$$

$$\psi_q = -e \sin \delta$$

From which there is, by obvious re-arrangement,

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$$i_d = \frac{E - e \cos \delta}{x_d} + e \frac{x_d - x_d(p)}{x_d x_d(p)} (\cos \delta_a - \cos \delta) + e \frac{x_d - x_d'}{x_d x_d'} \sum a_{2n} \epsilon^{-\alpha_{2n} t} \int_0^t \epsilon^{\alpha_{2n} u} \sin \delta(u) \delta'(u) du$$

$$i_q = \frac{e \sin \delta}{x_q} + e \frac{x_d - x_d(p)}{x_d x_d(p)} (\sin \delta_a - \sin \delta) + e \frac{x_d - x_d'}{x_d x_d'} \sum a_{2n} \epsilon^{-\alpha_{2n} t} \int_0^t \epsilon^{\alpha_{2n} u} \cos \delta(u) \delta'(u) du$$

$$T = \frac{E e \sin \delta}{x_d} + \frac{e^2 (x_d - x_d')}{2 x_d x_d'} \sin 2 \delta + e^2 \cos \delta \frac{x_d - x_d(p)}{x_d x_d(p)} (\sin \delta_a - \sin \delta) + e^2 \sin \delta \frac{x_d - x_d(p)}{x_d x_d(p)} (\cos \delta_a - \cos \delta) + e^2 \sin \delta \frac{x_d(p) - x_d G(p)}{x_d x_d(p)} \Delta E$$

$$T = \frac{E e \sin \delta}{x_d} + \frac{e^2 (x_d - x_d')}{2 x_d x_d'} \sin 2 \delta + e^2 \frac{x_d - x_d'}{x_d x_d'} \sin \delta \sum a_{2n} \epsilon^{-\alpha_{2n} t} \int_0^t \epsilon^{\alpha_{2n} u} \sin \delta(u) \delta'(u) du + e^2 \frac{x_d - x_d'}{x_d x_d'} \cos \delta \sum a_{2n} \epsilon^{-\alpha_{2n} t} \int_0^t \epsilon^{\alpha_{2n} u} \cos \delta(u) \delta'(u) du - \frac{e \sin \delta}{x_d} \sum b_n \epsilon^{-\beta_n t} \int_0^t \epsilon^{\beta_n u} \Delta E'(u) du$$

$$(49) \quad T = \frac{E e \sin \delta}{x_d} + \frac{e^2 (x_d - x_d')}{2 x_d x_d'} \sin 2 \delta + e^2 \frac{x_d - x_d'}{x_d x_d'} \sin \delta \sum a_{2n} \epsilon^{-\alpha_{2n} t} \int_0^t \epsilon^{\alpha_{2n} u} \sin \delta(u) \delta'(u) du + e^2 \frac{x_d - x_d'}{x_d x_d'} \cos \delta \sum a_{2n} \epsilon^{-\alpha_{2n} t} \int_0^t \epsilon^{\alpha_{2n} u} \cos \delta(u) \delta'(u) du - \frac{e \sin \delta}{x_d} \sum b_n \epsilon^{-\beta_n t} \int_0^t \epsilon^{\beta_n u} \Delta E'(u) du$$

$$(49a) \quad T = \frac{E e \sin \delta}{x_d} + \frac{e^2 (x_d - x_d')}{2 x_d x_d'} \sin 2 \delta + e^2 \frac{x_d - x_d'}{x_d x_d'} \sin \delta \sum a_{2n} \epsilon^{-\alpha_{2n} t} \int_0^t \epsilon^{\alpha_{2n} u} \sin \delta(u) \delta'(u) du + e^2 \frac{x_d - x_d'}{x_d x_d'} \cos \delta \sum a_{2n} \epsilon^{-\alpha_{2n} t} \int_0^t \epsilon^{\alpha_{2n} u} \cos \delta(u) \delta'(u) du - \frac{e \sin \delta}{x_d} \sum b_n \epsilon^{-\beta_n t} \int_0^t \epsilon^{\beta_n u} \Delta E'(u) du$$

But quantities $a_{2n}, a_{2n'}, \alpha_{2n}, \alpha_{2n'}, b_n, \beta_n$ may be found such that

$$\frac{x_d - x_d(p)}{x_d(p)} \cdot 1 = \frac{x_d - x_d'}{x_d'} \sum a_{2n} \epsilon^{-\alpha_{2n} t} \quad (50)$$

$$\frac{x_d - x_d(p)}{x_d(p)} \cdot 1 = \frac{x_d - x_d'}{x_d'} \sum a_{2n'} \epsilon^{-\alpha_{2n}' t}$$

$$\frac{x_d(p) - x_d G(p)}{x_d(p)} \cdot 1 = \sum b_n \epsilon^{-\beta_n t}$$

- $x_d' = x_d(\infty)$
- $x_d'' = x_d(\infty)$
- $\sum a_{2n} = 1.0$
- $\sum a_{2n'} = 1.0$
- $\sum b_n = 1.0$

It therefore follows from the operational rule that,

$$f(p) F(t) = F(0) \phi(t) + \int_0^t \phi(t-u) F'(u) du \quad (51)$$

where,

$$\phi(t) = f(p) \cdot 1$$

that if

- $\delta = \delta(t)$
- $p \delta = \delta'(t)$
- $\Delta E = \Delta E(t)$
- $p \Delta E = \Delta E'(t)$

Equations (48) and (49) may be rewritten in the form,

$$i_d = \frac{E - e \cos \delta}{x_d}$$

Formula (49a) may be used to determine starting torque and current with zero armature resistance, by introducing $\delta(t) = st, \delta'(t) = s$. Thus the average component of torque is found to be,

$$T_{av} = \frac{1}{2} \frac{x_d - x_d'}{x_d x_d'} \sum a_{2n} \frac{\alpha_{2n} s}{\alpha_{2n}^2 + s^2} + \frac{1}{2} \frac{x_d - x_d'}{x_d x_d'} \sum a_{2n'} \frac{\alpha_{2n'} s}{\alpha_{2n'}^2 + s^2} \quad (52)$$

Since

$$\frac{\alpha s}{\alpha^2 + s^2} \text{ is never greater than } \frac{1}{2}, \text{ and}$$

$$\sum a_{2n} = \sum a_{2n'} = 1.0$$

it follows that T_{av} is never greater than

$$\frac{1}{4} \left[\frac{x_d - x_d'}{x_d x_d'} + \frac{x_d - x_d'}{x_d x_d'} \right] \quad (53)$$

Equation (53) thus provides a very simple criterion of the maximum possible starting torque of a synchronous motor of given dimensions, when armature resistance is neglected.

The same formula may also be used to obtain a simple expression for the damping and synchronizing components of pulsating torque due to a given small angular pulsation of the rotor.

Thus if the angular pulsation is

$$\Delta \delta = [\Delta \delta] \sin(st)$$

and if the pulsation of torque is expressed in the form

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where,

$$T_{sa} = \frac{e I_{sa} \cos \delta_a}{x_d} + \frac{e^2 (x_d - x_d')}{x_d x_d'} \cos 2 \delta_a$$

δ_a = average angular displacement, i. e., total angle $= \delta = \delta_a + \Delta \delta$.

It can be shown that for the case of no additional rotor circuits, Equations (54) are exactly equivalent to Equations (24) and (25) in Doherty and Nickle's paper, *Synchronous Machines III*. The new formulas herein developed are, however, very much simpler in form, especially since in the case which Doherty and Nickle have treated, there is only one term in the summation; that is, $n = 1$, and α is merely the reciprocal of the short circuit time constant of the machine, expressed in radians.

K. The Equivalent Circuit of Synchronous Machines Operating in Parallel at No Load, Neglecting the Effect of Armature Resistance

Let, δ_a = angle of rotor *a* and bus
 θ_a = angle of rotor *a* in space

In general, the shaft torque of a machine depends on its acceleration and speed in space, and the magnitude and rate of change of the bus voltage as a vector. If all of the machines are operating at no load and if there is no armature resistance, a small displacement of any one machine will change the magnitude of the bus voltage only by a second order quantity; consequently for small displacements the magnitude of the bus voltage may be regarded as fixed, and only the angle of the bus and rotor need be considered. Furthermore, the electrical torque may be found in terms of (δ) by employing an infinite bus formula. But Equation (49a) implies the alternative general operational form,

$$T = \frac{e I_{sa} \sin \delta}{x_d} + \frac{e^2 (x_d - x_d') \sin 2 \delta}{2 x_d x_d'} - \frac{x_d - x_d'}{x_d x_d'} e^2 \sin \delta \sum \frac{a_{sa} p}{p + \alpha_{sa}} \cos \delta \quad (49b)$$

$$+ \frac{x_d - x_d'}{x_d x_d'} e^2 \cos \delta \sum \frac{a_{sa} p}{p + \alpha_{sa}} \sin \delta$$

$$T_{sa} = \left[\frac{e I_{sa}}{x_d} + e^2 \frac{(x_d - x_d')}{x_d x_d'} \right] \delta_a$$

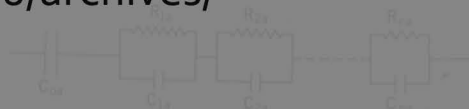


FIG. 8

voltage across the circuit represents the electrical torque of the machine (T_{sa}).

The capacitances and resistances must be chosen so that

$$C_{sa} = \frac{x_{d0} x_{d0}'}{e I_{sa} x_{d0} + e^2 (x_{d0} - x_{d0}')} \quad (56)$$

$$C_{sa} = \frac{x_{d0} x_{d0}'}{e^2 a_{sa} (x_{d0} - x_{d0}')}$$

$$R_{sa} = \frac{1}{C_{sa} \alpha_{sa}}$$

The equation for the mechanical torque is

$$T_{sa} = T_a + M_a p s_a \quad (57)$$

where:

M_a = inertia factor of machine *a* in radians

$$= \frac{2 \times \text{stored mech. energy at normal speed}}{\text{base power}}$$

$$= 2 \pi f \frac{0.462 W R^2 \left(\frac{\text{rev. per min.}}{1000} \right)^2}{\text{base kw.}}$$

s_a = per-unit speed of machine *a*

$$t = \text{time in seconds} \left(p = \frac{d}{dt} \right)$$

But,

$$s_a = p \theta_a$$

Thus there is

$$T_{sa} = T_a + M_a p^2 \theta_a \quad (57a)$$

which corresponds to the equivalent circuit of Fig. 9, in which change $= \theta_a$

$$L_a = M_a$$

The machine operating on an infinite bus can be

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represented by the equivalent circuit of Fig. 10, since in the inductive branch of the circuit. Thus a governor which acts through a single time constant may be

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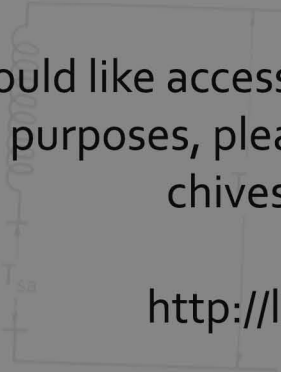


Fig. 9

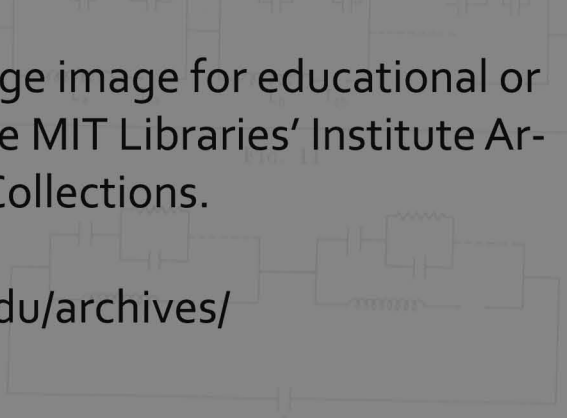


Fig. 12

represented by the diagram of Fig. 11, since the conditions

$\theta_s - \delta_s = \theta_r - \delta_r = \dots$ (= bus angle in space)
 $T_s + T_r + T_{\dots}$, etc. = bus power output = 0

A transmission line may be represented by a condenser.

Thus two machines connected by a line of reactance (x) would be represented by the circuit of Fig. 12, where

$$C = \frac{x}{e^2} \quad (58)$$

Shaft torques are, of course, represented by voltages.

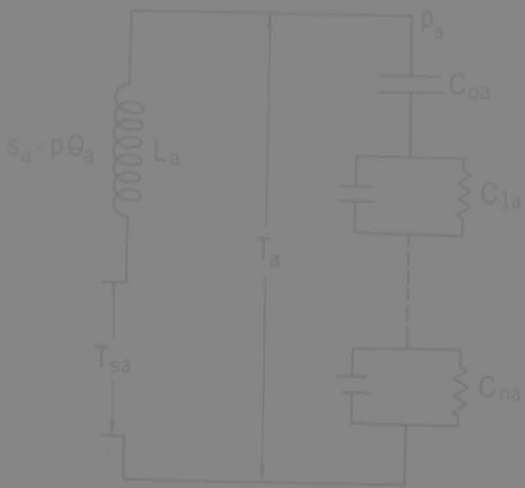


Fig. 10



Fig. 13

Mechanical damping, such as that due to a fan on a motor shaft or that due to the prime mover, is represented by resistance in series with the inductance (L) as in Fig. 13. (R) must be chosen equal to the rate of decrease in available driving torque with increase in speed.

Governors and other prime mover characteristics may also be represented by connecting their circuits

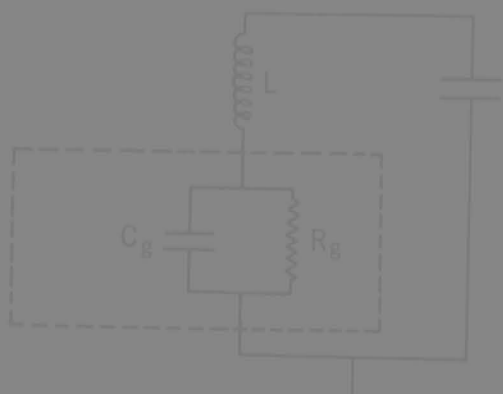


Fig. 14

$$R_v = \frac{1}{\text{regulation}}$$
$$C_v = \frac{\text{time constant of governor in elec. radians}}{R_v} \quad (59)$$

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Mar 1, 1930
H. E. Edgerton

The exponent $e^{j\theta}$ can be thought of as an operator which takes into account the speed of the rotor. For instance, in the first of the four equations, the voltages are in terms of drop with respect to the stator currents. The current in the rotor, of slip frequency for an induction motor in the steady state, is speeded up by a frequency of $(1-s)$, so that its apparent frequency from the stator side is that of the fundamental. Likewise in the expressions for the rotor voltage, the stator currents have their frequency reduced to that of the rotor.

STEADY STATE

BALANCED STATOR AND ROTOR. BAL. VOLTAGE.

With balanced voltage on the stator we know that $E_1 = e_{+1} + e_{-1}$ and $E_2 = -j e_{+2} + j e_{-2}$, or solving for the components.
 $e_{+1} = \frac{E_1 + j E_2}{2}$ and $e_{-1} = \frac{E_1 - j E_2}{2}$. Also the rotor will run at a constant speed, i.e. θ has a const. rate of increase.

$\theta = (\omega - n)t$ n = electrical angular velocity of the rotor. In the steady-state it is known that the current in both the rotor and the stator are sinusoids of frequencies determined by the applied potential E and the slip $[s = \frac{\omega - n}{\omega}]$. Thus the components of current are of the form:

$i_{+2}' = \frac{I_2}{\sqrt{2}} e^{j(\omega-n)t}$ $i_{-2}' = \frac{I_2}{\sqrt{2}} e^{-j(\omega-n)t}$ When the rotor is shorted $E_{+2} = E_{-2} = 0$.

$i_{+1}' = I_1 e^{j\omega t}$ $i_{-1}' = I_1 e^{-j\omega t}$

This in the differential equations for the + components;

$e_{+1} = \frac{E_1}{\sqrt{2}} = Z_1 \frac{I_1}{\sqrt{2}} e^{j\omega t} + M p \left(\frac{I_2}{\sqrt{2}} e^{j(\omega-n)t} \right)$ $Z_1 = (r_1 + j\omega L_1)$

$e_{-1} = 0 = Z_2 \frac{I_2}{\sqrt{2}} e^{-j(\omega-n)t} + M p \left(\frac{I_1}{\sqrt{2}} e^{j\omega t} \right)$ $Z_2 = (r_2 + j\omega s L_2)$

cancel exponentials

$E_1 = Z_1 I_1 + j\omega M I_2$ $(\omega - n) = s\omega$

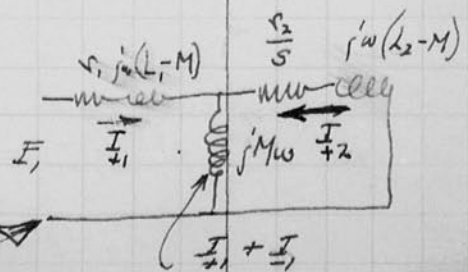
$0 = Z_2 I_2 + j\omega M s I_1$ $s = \frac{(\omega - n)}{\omega}$

add and subtract

$E_1 = [r_1 + j\omega(L_1 - M)] I_1 + j\omega M (I_2 + I_1)$

$0 = [r_2 + j\omega s(L_2 - M)] I_2 + j\omega M (I_1 + I_2)$

Which gives the ordinary steady state equations for the ind. motor and the equivalent circuit.



Ind. Motor with ϕ Rotor.Mar. 1, 1930
J. E. Edgerton

A three phase induction motor with a ~~single~~ phase rotor circuit has a differential equation that is given by the following component equations. (This is given in Ku's paper A.I.E.E. 1929 Jan-Feb).

$$(r_1 + L_1 p) i_1' + \frac{M}{2} p i_2' \varepsilon^{i n t} = v_1 \quad (1)$$

$$(r_2 + L_2 p) i_2' + \frac{M}{2} p i_1' \varepsilon^{-i n t} = 0 \leftarrow \text{Bal. volts.} \quad (2)$$

$$(r_2 + L_2 p) i_2' + \frac{3}{2} M p (i_1' \varepsilon^{-i n t} + i_1' \varepsilon^{i n t}) = 0. \quad (3)$$

Proceeding as before to pick out the steady state solutions, we select the form of i_2' since we know that it will be sinusoidal. The magnitude I_2 is undetermined but the frequency is that of the slip.

$$i_2' = I_2 \left(\frac{\varepsilon^{i \omega t s} + \varepsilon^{-i \omega t s}}{2} \right) \quad s = \text{slip.}$$

When this is used in (1) it is found that i_1' must have two frequencies of components, one of rated frequency (that of v_1) and the other of $(1-2s)$ or $(2\pi - \omega)$.

From (1)

$$(r_1 + L_1 p) i_1' + \frac{M}{2} I_2 \left[\frac{j \omega \varepsilon^{i \omega t s} - j \omega \varepsilon^{-i \omega t s}}{2} \right] \varepsilon^{i n t}$$

$$(r_1 + L_1 p) i_1' = v_1 - \frac{M}{2} p I_2 \left(\varepsilon^{i(\omega s + n)t} + \varepsilon^{-i(\omega s - n)t} \right) \quad \begin{matrix} \omega - n - n = \omega - 2n \\ (\omega - n + n) = \omega \end{matrix}$$

$$i_1' \text{ is of the form } I_A \varepsilon^{i \omega t} + I_B \varepsilon^{i(2\pi - \omega)t} \\ (r_1 + L_1 p) i_1' = E \varepsilon^{i \omega t} - \frac{M I_2}{4} \left(j \omega \varepsilon^{i \omega t} + j(2\pi - \omega) \varepsilon^{i(2\pi - \omega)t} \right)$$

Equating each frequency

$$(r_1 + j \omega L_1) I_A \varepsilon^{i \omega t} = E \varepsilon^{i \omega t} - \frac{M I_2}{4} j \omega \varepsilon^{i \omega t}$$

$$(r_1 + j(2\pi - \omega) L_1) I_B \varepsilon^{i(2\pi - \omega)t} = -j(2\pi - \omega) \frac{M I_2}{4} \varepsilon^{i(2\pi - \omega)t}$$

Cancelling exponential

$$Z_1 I_A = E - j \omega \frac{M I_2}{4}$$

$$Z_2 I_B = -j(2\pi - \omega) \frac{M I_2}{4}$$

In a similar manner the expressions for the steady state values of i_2 and i_4 can be put into equation (3), thus determining the form of equation that i_1 must have.

From equation (3):

$$(r_2 + j\omega L_2)I_2 + \frac{3}{2}M\omega \left[I_A e^{j\omega t - n\tau} + I_B e^{j(2n-\omega)t - m\tau} \right] + \frac{3}{2}M\omega \left[I_C e^{j\omega t} + I_D e^{j(2n-\omega)t} \right] = 0$$

$$I_2 [r_2 + j\omega L_2] e^{j\omega t} + I_2 (r_2 + j\omega L_2) e^{-j\omega t} + \frac{3}{2}Mj\omega S I_A e^{j(\omega-n)t} + \frac{3}{2}M(-j\omega S) I_B e^{-j(\omega-n)t} + \dots = 0$$

$i_1 = I_C e^{-j\omega t} + I_D e^{-j(2n-\omega)t}$

Separating the different freq. and cancelling ~~freq.~~ exponentials

$$I_2 [r_2 + j\omega L_2] e^{j\omega t} + \frac{3}{2}Mj\omega S I_A e^{j\omega t} + \frac{3}{2}j\omega S M I_D e^{j\omega t} = 0$$

$$\text{and } I_2 [r_2 + j\omega L_2] e^{-j\omega t} + \frac{3}{2}Mj\omega S I_B e^{-j\omega t} - \frac{3}{2}j\omega S M I_C e^{-j\omega t} = 0$$

$$I_2 r_2 + jX I_A + jX I_D = 0$$

$$I_2 (r_2) - jX I_B + jX I_C = 0$$

I_A see below

I_B ?

Equation (2) with known value of i_2 from assumption of the steady state.

$$(r_1 + L_1 p) i_1 + \frac{M}{2} \frac{I_2}{2} \left(\frac{e^{j\omega t - n\tau} + e^{-j\omega t - n\tau}}{2} \right) = 0$$

$\frac{e^{j(\omega-2n)t} + e^{-j\omega t}}{2}$

$$(r_1 + L_1 p) i_1 + \frac{MI_2}{4} \left(-j\omega e^{-j\omega t} - j(2n-\omega) e^{-j(2n-\omega)t} \right) = 0$$

i_1 must have two frequencies and be of the form.

$$i_1 = I_A e^{-j\omega t} + I_B e^{-j(2n-\omega)t} \quad \text{since also } i_4 + i_1 = \text{real} = i_1'$$

Equating equal freq. and cancelling exponentials.

$$(r_1 - j\omega L_1) I_A - \frac{MI_2}{4} j\omega = 0$$

$$(r_1 - j(2n-\omega) L_1) I_B - \frac{MI_2}{4} j(2n-\omega) = 0$$

Collecting various vector expressions.

$$I \begin{cases} Z_1 I_A = E - j\omega \frac{MI_2}{4} \\ Z_{11} I_B = -j \frac{MI_2}{4} (2\eta - \omega) \end{cases}$$

From + sequence

$$II \begin{cases} Z_{10} I_A = j\omega \frac{MI_2}{4} \\ Z_{10} I_B = j(2\eta - \omega) \frac{MI_2}{4} \end{cases}$$

don't need this.

$$III \quad Z_2 I_2 + j \frac{3}{2} M \omega s [I_A + I_B] = 0$$

From I and III.

$$I_2 = \frac{-E j \frac{3}{2} M \omega s / Z_1}{\left[Z_2 + \frac{3}{8} \frac{M^2 \omega^2 s}{Z_1} + \frac{3}{8} \frac{M^2 \omega (2\eta - \omega) s}{Z_{11}} \right]}$$

$$= \frac{-E j \frac{3}{2} M \omega s / Z_1}{Z_2 + \frac{3}{8} M^2 \omega^2 s \left[\frac{1}{Z_1} + \frac{2\eta - \omega}{\omega} \frac{1}{Z_{11}} \right]}$$

$$\begin{aligned} \frac{2\eta - \omega}{\omega} &= 1 - 2s \\ \text{since} \quad \frac{2\eta - 2\omega + \omega}{\omega} &= \frac{\omega - 2(\omega - \eta)}{\omega} \\ &= 1 - 2s. \end{aligned}$$

$$\frac{1}{Z} = j \left[\frac{2Z_1 Z_2}{3 \sin M} + \omega M \left(\frac{1}{4} + \frac{1}{4} (1-2s) \frac{Z_1}{Z_{11}} \right) \right]$$

$$\frac{-E j \frac{3}{2} M \omega s / Z_1}{Z_2 + \frac{3}{8} M^2 \omega^2 s \left[\frac{Z_{11} + (1-2s) Z_1}{Z_1 Z_{11}} \right]}$$

$$\begin{aligned} Z_1 &= r_1 + j\omega L_1 \\ Z_{11} &= r_1 + j(2\eta - \omega) L_1 \\ &= r_1 + j\omega L_1 (1-2s) \end{aligned}$$

$$I_A = \frac{E - j\omega \frac{M}{4} \left(\frac{+E}{Z_2} \right)}{Z_1} = \frac{E \left(1 - j \frac{\omega M}{4 Z_2} \right)}{Z_1} = \frac{E}{Z_A}$$

$$I_B = -j \frac{M}{4} \frac{(2\eta - \omega)}{Z_{11}} \left(\frac{+E}{Z_2} \right) = -j \frac{M}{4} \frac{(1-2s)\omega}{Z_{11}} \frac{E}{Z_2} = \frac{-j \omega M (1-2s) E}{4 Z_{11} Z_2} = \frac{E}{Z_B}$$

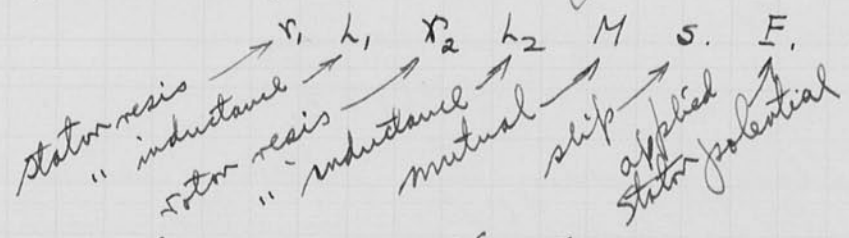
$$Z_A = \frac{Z_1}{1 - j \frac{\omega M}{4 Z_2}}$$

$$Z_B = \frac{Z_{11}}{-(1-2s) j \frac{\omega M}{4 Z_2}} =$$

Mar 21 1930
J. E. Edgerton

Currents an Ind. Motor with a 1 phase rotor.

In the preceding three pages, I have obtained expressions for the magnitudes, (vector) of the current in the rotor and the two components of current in the stator. This ~~is~~ was done by assuming the current in the rotor to be a sinusoidally varying one, the magnitude being unknown. By inspection of the differential equations, it was found that two current components must exist in the stator, one of fundamental frequency and one of $(1-2s)$ fund. freq. This was also known experimentally, but it could be deduced from the differential equations. Now the various frequencies ~~are~~ of currents were given unknown magnitudes and placed in the differential equations for the steady state. In this case the p (or $\frac{d}{dt}$) is replaced by the $j\omega$ or $j(2\pi - \omega)$ for the $(1-2s)$ frequency. In this manner three equations are obtained with three unknowns after the different frequency components are equated and the exponentials cancelled. The three unknowns (I_2 , I_A , and I_B) are then solved in terms of



L_1 = synchronous self inductance of stator per phase. It is the self inductance of one phase (stator), plus the mutual effect of each of the other stator phases.

L_2 = self inductance of the ~~field~~ rotor.

The current in the rotor is then:

$$i_2' = \frac{I_2 \cos \omega s t}{\sqrt{2}} = \frac{E \cos \omega s t}{Z_2} \quad \text{where } Z_2 = j \left[\frac{2}{3} \frac{Z_1 Z_2}{s \omega M} + \omega M \left(\frac{1}{4} + \frac{1}{4} (1-2s) \frac{Z_1}{Z_2} \right) \right]$$

$$Z_1 = r_1 + j\omega L_1$$

$$Z_2 = r_2 + j\omega L_2$$

$$Z_{11} = r_2 + j\omega L_2 (1-2s)$$

The stator current in phase one is equal to

$$\begin{aligned} i_1' &= i_4' + i_2' \\ &= 2 \left[I_A \cos \omega t + I_B \cos(2\pi - \omega)t \right] \\ &= 2 \left[I_A \cos \omega t + I_B \cos \omega(1-2s)t \right] \end{aligned}$$

And the rotor current as demonstrated on the previous page is

$$i_2 = I_2 \cos s\omega t.$$

In these expressions I_2 , I_A , and I_B are vectors which are determined by the constants of the motor that is under question (and the slip).

$$I_2 = \frac{E}{Z_2} \quad Z_2 = j \left[\frac{2Z_1 Z_2}{3s\omega M} + \omega M \left[\frac{1}{4} + \frac{1}{4}(1-2s) \frac{Z_1}{Z_{11}} \right] \right]$$

$$I_A = \frac{E}{Z_A} \quad Z_A = \frac{Z_1}{1 - j \frac{\omega M}{4Z_2}}$$

$$I_B = \frac{E}{Z_B} \quad Z_B = \frac{Z_{11}}{-(1-2s)j \frac{\omega M}{4Z_2}}$$

Needless to say the above calculations are quite difficult to make since they involve quite a few vector expressions.

Major oral exam in Elect Eng. Apr. 2, 1930 18-200

Notebook # 3

Filming and Separation Record

 unmounted photograph(s)

 negative strip(s)

 3 unmounted page(s)
(notes, drawings, letters, etc.)

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Item(s) now housed in accompanying folder.

Equation for torque

$$T = P j' M \left(z'_{a2} z'_{b1} \Sigma^{j\theta} - z'_{a1} z'_{b2} \Sigma^{j'\theta} \right)$$

with the single phase rotor.

$$z'_{b1} + z'_{b2} = I_b \cos s\omega t = I_b \frac{\Sigma^{j\omega t} + \Sigma^{-j\omega t}}{2}$$

$$-j'z'_{b1} + j'z'_{b2} = 0$$

$$z'_{b1} = \frac{I_b}{2} \cos s\omega t$$

$$z'_{b2} = \frac{I_b}{2} \cos s\omega t$$

$$\begin{aligned} (2n - \omega - \omega) + \omega \\ 2n - \omega \\ \omega - -s\omega + \omega \\ \omega - 2s. \end{aligned}$$

$$\begin{aligned} -j2n + j\omega + j\omega - j's \\ -j2(\omega - n) - j's \\ -j2s - j's \\ -j'3s. \end{aligned}$$

$$T = j' P M \left(z'_{a2} \Sigma^{j\theta} - z'_{a1} \Sigma^{-j'\theta} \right) \frac{I_b}{2} \cos s\omega t$$

$$z'_{a1} = I_A \Sigma^{j\omega t} + I_B \Sigma^{+j(2n-\omega)t}$$

$$z'_{a2} = I_A \Sigma^{-j\omega t} + I_B \Sigma^{-j(2n-\omega)t}$$

$$\theta = (\omega - s)t$$

$$T = j' P M \left(I_A \Sigma^{-j\omega t} \Sigma^{j(\omega-s)t} + I_B \Sigma^{-j(2n-\omega)t} \Sigma^{j(\omega-s)t} \right)$$

$$= \text{conj} \left(\frac{I_b}{2} \cos s\omega t \right)$$

$$= j' P M \left(I_A \Sigma^{-j'st} + I_B \Sigma^{-j'3st} \right) \frac{I_b}{2} \cos s\omega t$$

$$T = \text{real} \times 2$$

PULLING INTO STEP OF A SALIENT-POLE SYNCHRONOUS
MOTOR UNDER LIGHT LOAD CONDITIONS.

A salient-pole motor when brought up to speed as an induction motor usually pulls into synchronism and operates as a reluctance motor, if the shaft load is small and other conditions favorable. The salient-poles follow the rotating m.m.f. of the armature by an angle sufficient to supply the load on the shaft. This reluctance torque as a function of the angle is a $\sin 2\theta$ term. The reason that it depends upon the double angle is because the polarity of the salient poles is not definite but depends upon the position. In other words the salient pole is a mass of iron that endeavors to place itself in a place where the maximum flux will exist and it does not depend on whether the field is north or south.

The synchronous torque expressions for the salient-pole machine as given by Doherty and Nickle at the A.I.E.E. convention in June 1926, are

$$P = \frac{EV x_d}{z^2} \sin \theta + \frac{V^2 (x_d - x_q)}{z^2} \sin 2\theta + \frac{rV}{z^2} (V - E \cos \theta).$$

The first term ($\sin \theta$) is the synchronizing power due to the field current. E is the induced e.m.f. in the stator. V is the applied potential. x_d is the reactance in the direct axis. $z^2 = \frac{r^2 + x_d^2 x_q}{x_d - x_q}$ where x_q = the reactance in the quadrature axis and r = the armature resistance.

The second term ($\sin 2\theta$) is the reluctance power. The last term corrects for the losses that exist in the armature. The sum of these components equals the input to the motor.

Before the field current is connected the input to the motor is equal to

$$P = \frac{V^2 (x_d - x_q)}{z^2} \sin 2\theta + \frac{r V^2}{z^2}$$

Core loss has been neglected here. In case measurements can be

Studied in Apr 21 1930. H.E.E.

made then the core loss should be subtracted from P before it is equated to the other quantities.

$$P - (\text{core loss}) = \frac{V^2 (x_d - x_q)}{z^2} \sin 2\theta + \frac{r V^2}{z^2}$$

The shaft load being small (only windage and friction), $\sin 2\theta$ will be small and satisfies the equation in four positions for each 360 electrical degrees of displacement. Two of these angles are unstable (90 and 270 degrees) since the slope of the torque curve is negative. The two possible operating angles are 0 and 180 degrees. When the motor pulls into step as a reluctance motor it may be at either of these angles (0 or 180 degrees.)

Thus when the field is connected to a supply of d-c. there are three possibilities;

- (1) the motor will operate at zero angular displacement but with a much smaller armature current.
- (2) The motor will operate at 180 degrees of angular displacement but with increased armature current.
- (3) The motor will not operate continuously as case (2) if the field current is large enough. In this case the motor must slip a pole, resulting in violent pulsations of current and power.

The three attached oscillograms were taken for the three cases that are listed above.

December 18th, 1929.

H. E. Edgerton.

Massachusetts Institute of Technology.

Notebook # 3

Filming and Separation Record

___ unmounted photograph(s)

___ negative strip(s)

1 unmounted page(s)
(notes, drawings, letters, etc.)

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Nov 15

Machine 804 A + B

With only Wind & Friction on set + small machine drawing

Watts	Vt	Ia	RPM
1580	220	4.18	1204
1520	218	4.1	1204

With large machine excited for core loss.

Wattoutput	Vt _{804B}	Ia	If _{804A}	RPM	Ea _{804A}
2020	220	5.34	7.3	1210	211
2160	219	5.64	9.2	1208	244
2320	218.5	6.05	10.65	1202	266
2528	217.5	6.70	12.5	1201	287
2980	215.0	7.90	16.0	1196	320

Short circuit on 804 A.

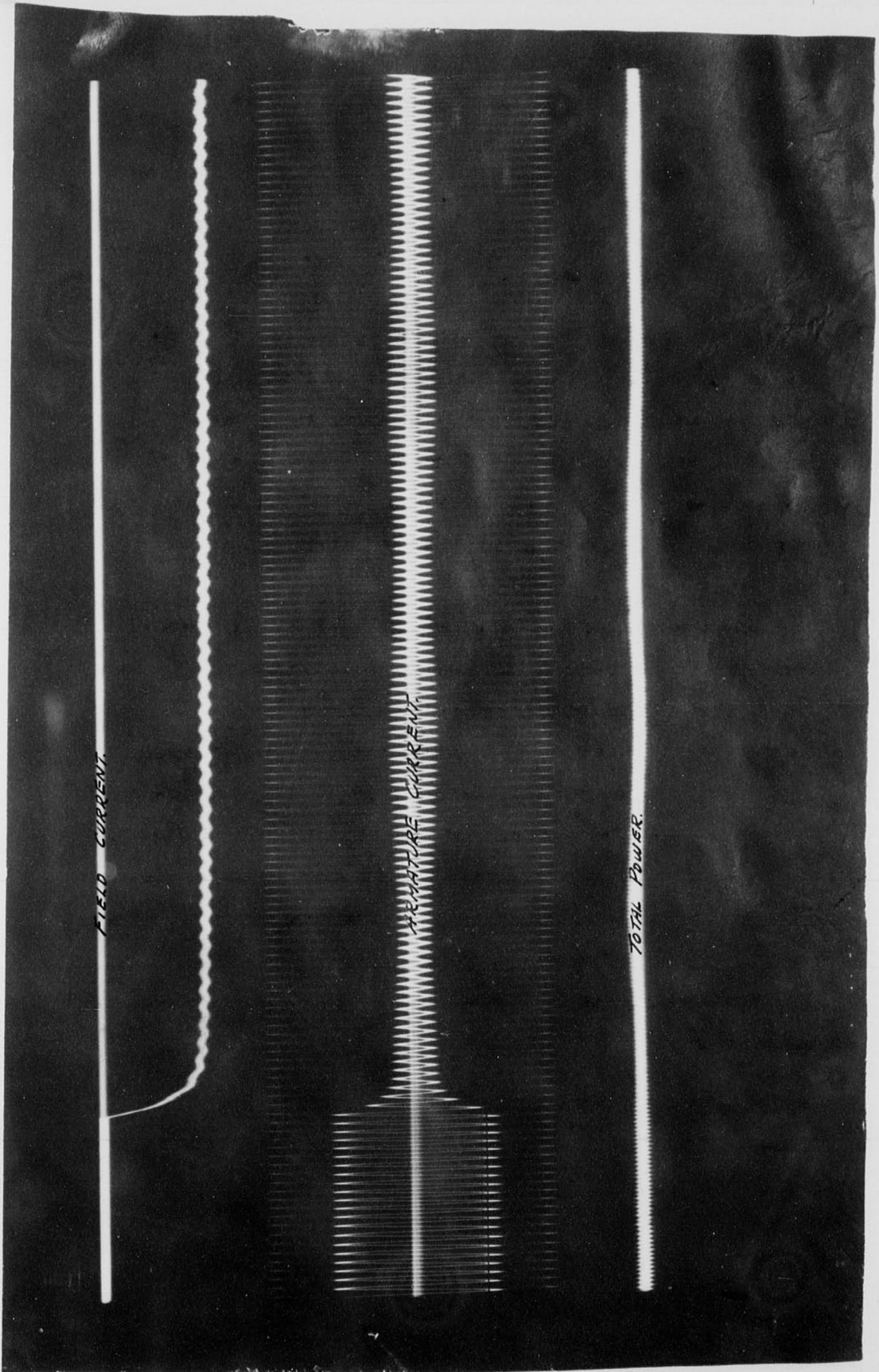
Wattoutput	Vt _{804B}	Ia _{804B}	If _{804A}	Speed	Ia _{804A}
1880	218	5	5.05	1204	76.8
2840	216.8	7.5	9.25	1205	141
2140	217.5	5.65	6.50	1205	99.2
3600	2170	9.6	11.45	1206	174
1440	217.5	3.85	0	1202	—

With added reactance on 804 A & short circuited

If	Ia1	Ia2	Ia3	RPM	Ea to N	Xs	
4.6	22	21.8	21.5	1200	79	3.62	iron reactor
7.7	37	36.5	36.2		129.5	3.54	
10.1	47.6	46.5	46.1		152	3.26	
12.3	55.9	54.3	54.1		167.5	3.06	
4.15	51.2	50.8	49.9		72	1.42	air core reactor
5.20	64	63.7	62.8		89.5	1.41	
6.40	78.4	77.8	77.3		108.5	1.395	

[Signature]

Use. No. 1.

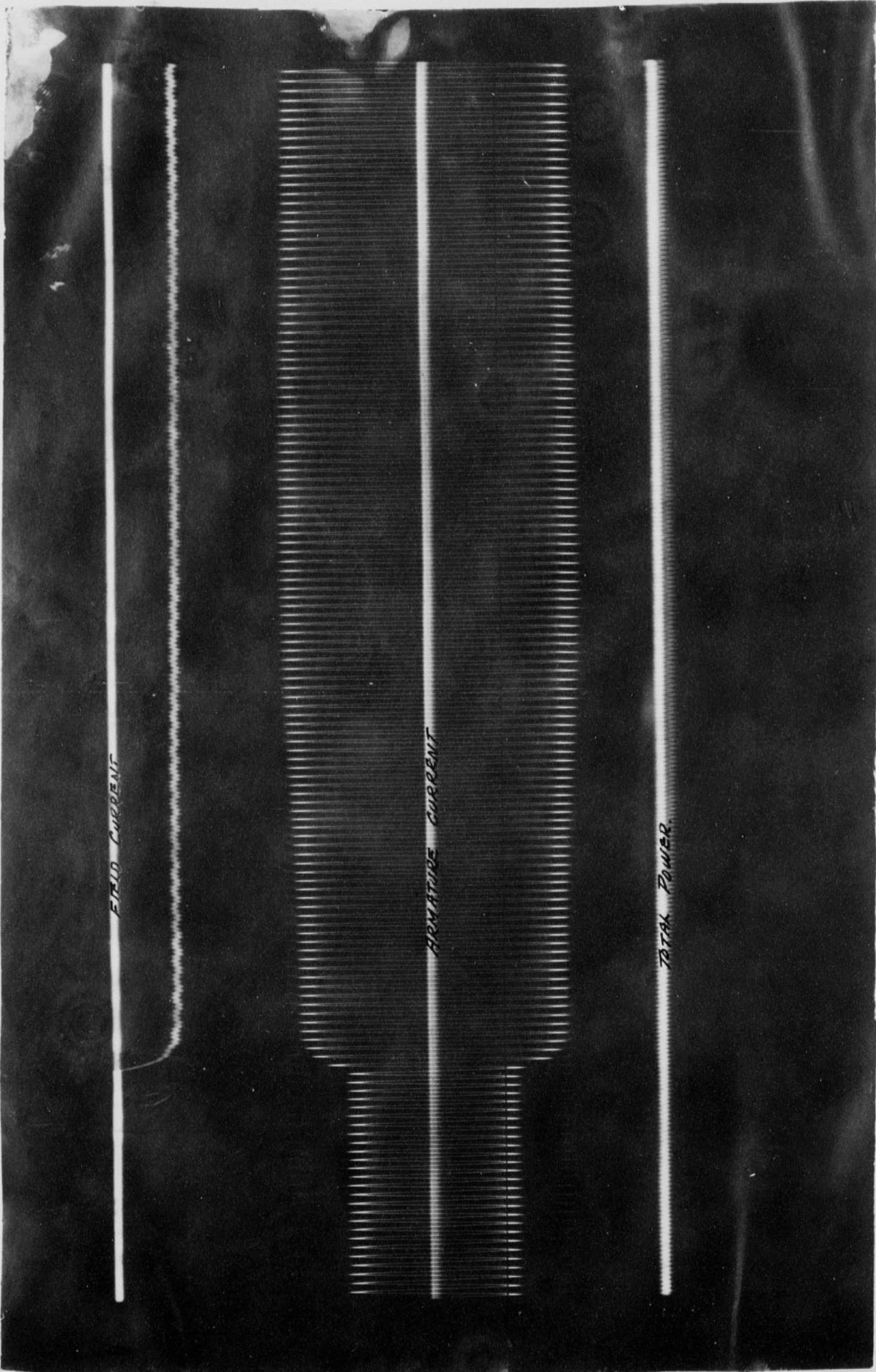


FIELD CURRENT

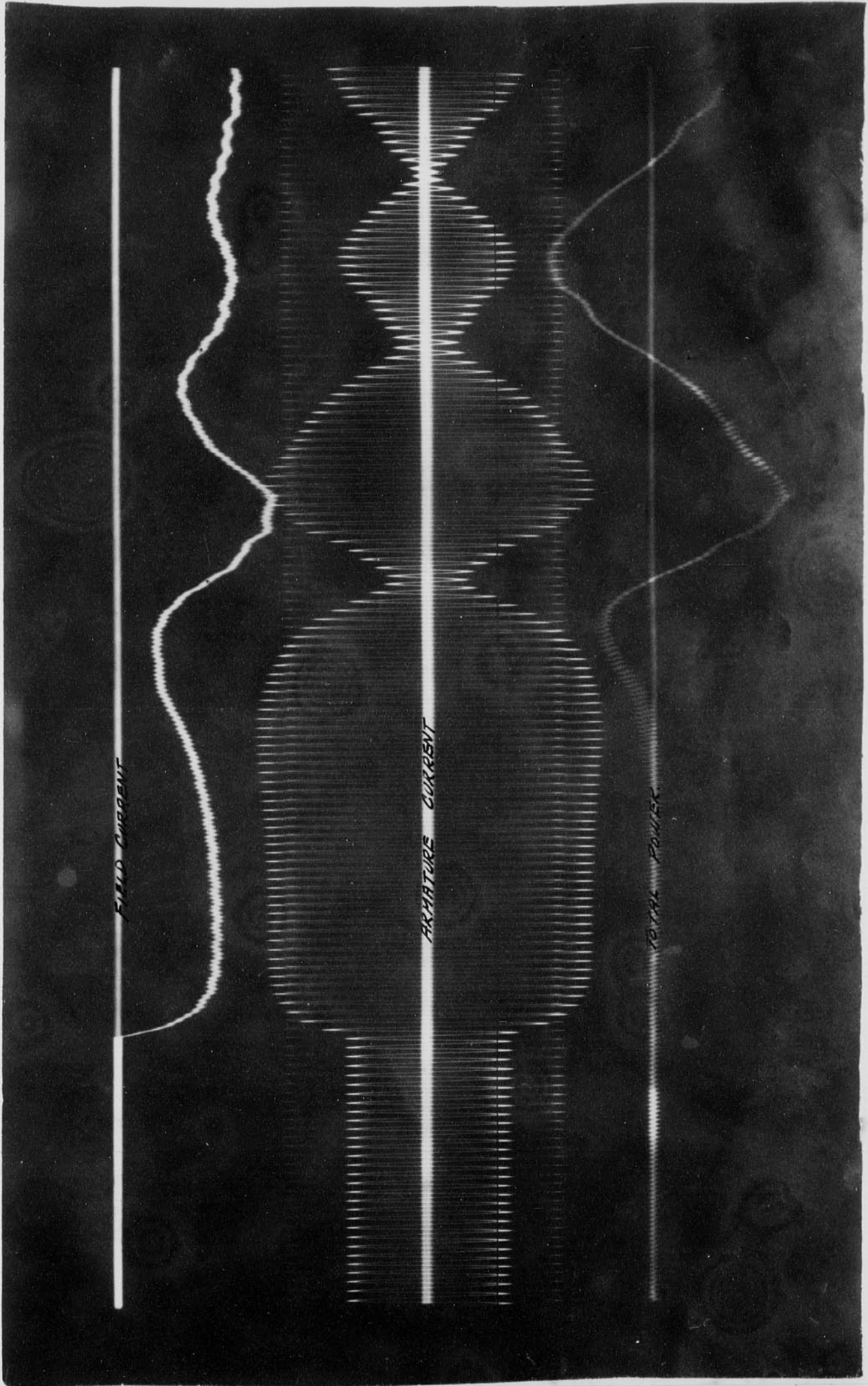
HEAT TUBE CURRENT

TOTAL POWER

Osc. No. 1.



Osc. No. 2



Osc. No. 3.

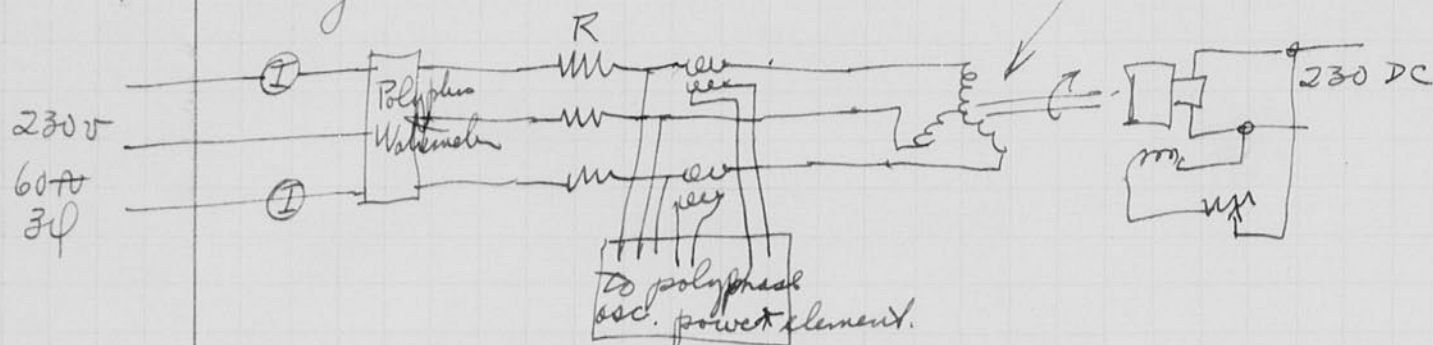
May 2, 1930.

L. E. Edgerton

Resistance of leads for measuring resistance
 $= 0.153$ ohms. loose contact.
 $R = 0.129$ ohms.

May 3, 1930

L. E. Edgerton



When R was about 0.7 of an ohm the damping was small but still positive.

$$R_1 = 1.46 - .13 = 1.33 \text{ ohms.}$$

$$R_2 = 1.82 - .13 = 1.69$$

$$R_3 = 1.62 - .13 = 1.49$$

With this resistance the motor has very little damping with $E = V$. Excitation on fld 1 with 110 volts as source.

Could not show neg power running as ind motor with 1 of rotor.

$$R_1 = 3.23 - .13$$

$$R_2 = 2.88 - .13$$

$$R_3 = 2.49$$

Osc No 7. No excitation 1 of rotor. (Regular field winding short-circuited).
 D.C. machine delivering power into lines.
 The rotor current vibrator was sticking

Osc No. 8. Repeat. Hazards for V_{range} and V_{speed} .

Osc No 9, calib reduced on Vibrator #3 (rotor amps).

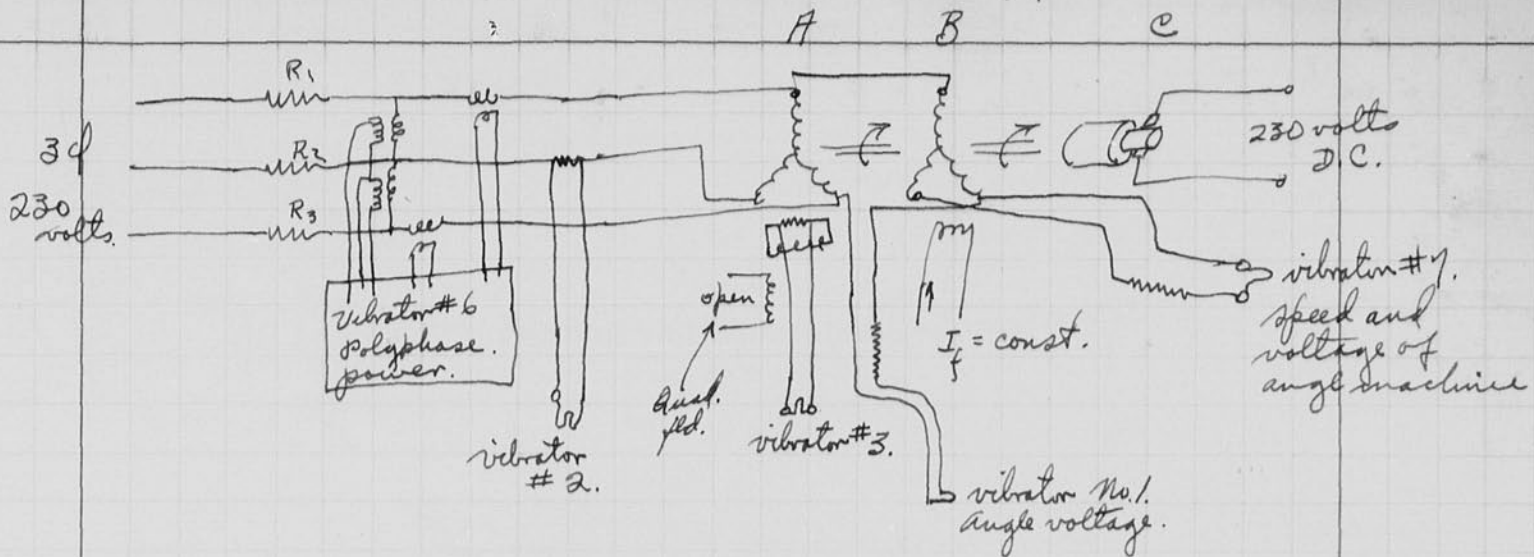
$$\text{Hot. } R_1 = 2.29 - .13 = 2.16$$

$$R_2 = 2.40 - = 2.27$$

$$R_3 = 2.32 - = 2.19$$

$$\frac{3 \overline{6.62}}{2.21 \text{ ohms avg.}}$$

Set 99



Osc.	H.W	I_1	I_2	V	I_f	E.	
10	547400	656691	656760	24873		24873	R_1, R_2 and R_3 shorted for calib. run.
Calibration	$\times 4$ 19.2	19.0	20.7	228		352	Fld No 1 (West)
	$\times 2$ 5.6	$3 \pm$	$3 \pm$	228		226	Fld No 1 (West)

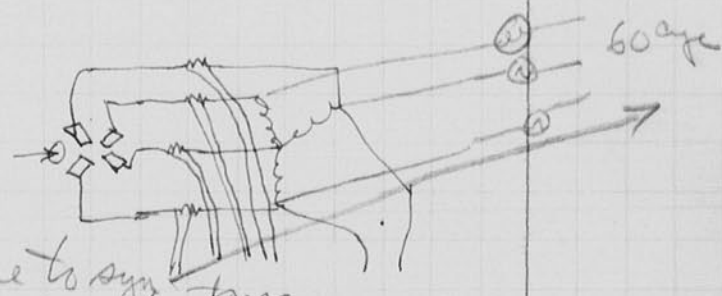
Winding Friction (core loss of 99 A and 99 B).

Failed to record the zero line of the power as same as film No. 9.

Osc	Self oscillation. Qual fld open.			
11.	the Power element put ahead of the resistances.		230.	Zero of P-Power shifted. other calib same.
12.	—	—	234	220

13 S.S. test on new switch

V	I	
98.	9.96 9.1	slow osc. es due to sym arrangement, loose.



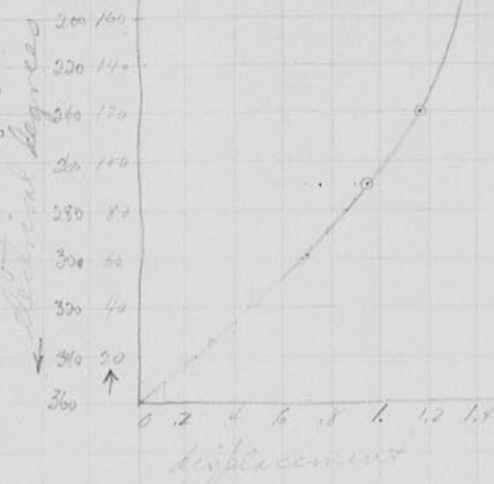
14. Same except time delay relay set on fastest point.

Calculation of Oscillograms.

May 4, 1930
S. E. EgertonFrom film no 9. when $\theta = 180$ electrical degrees

$$E_1 = \frac{135}{180} \text{ inches}$$

P inch	Sec.	E_1 inch	P inch	I inch	θ	P kW.	KVA.
-39	1	-23	.60	.20	-20	-4.61	2.97
1.09	2	+0.72	.39	.09	6	-3.0	1.33
.71	3	.40	+0.6	.36	35	+4.46	5.35
1.25	4	.65	+1.71	.60	58	5.46	8.92
1.49	5	.78	1.25	.90	70	9.6	10.4
1.57	6	.86	1.49	.70	79	11.45	10.4
1.41	7	.99	1.52	.65	71	11.7	9.65
1.15	8	.68	1.41	.53	61	10.8	7.87
.68	9	.40	1.15	.32	35	8.83	4.75
.09	10	+0.68	.68	.09	7	5.225	1.04
-39	11	-.27	+0.09	.20	-19	-6.92	2.97
-60	0	-.45	-.39	.39	-36	-3.0	5.5



From osc. 10

1 inch of Poly Power
= 7.68 KW.

0.5 inch = 18.8 amp or 7.42

1 inch = 4.84 kva.

Calc of Evans and Sels chart.

$$V = 228$$

$$R = 2.21 + 0.3^2 = 2.54 \text{ ohms per phase}$$

$$X = 4.53 \text{ ohms } \pm$$

$$\frac{V_0}{V_0} = 0.14 \times 2$$

$$Z = \sqrt{2.54^2 + 4.53^2} = 5.2 \text{ ohms.}$$

$$P_{ed} = \frac{228^2}{5.2 \times 1000} = 10.1 \text{ kw}$$

$$P = \frac{V^2 R}{Z^2} = \frac{228^2 \times 2.54}{(1000) \times 27} = 4.79 \text{ Kw}$$

$$Q = \frac{V^2 X}{Z^2} = \frac{228^2 \times 4.53}{27} = 8.73 \text{ Kw.}$$

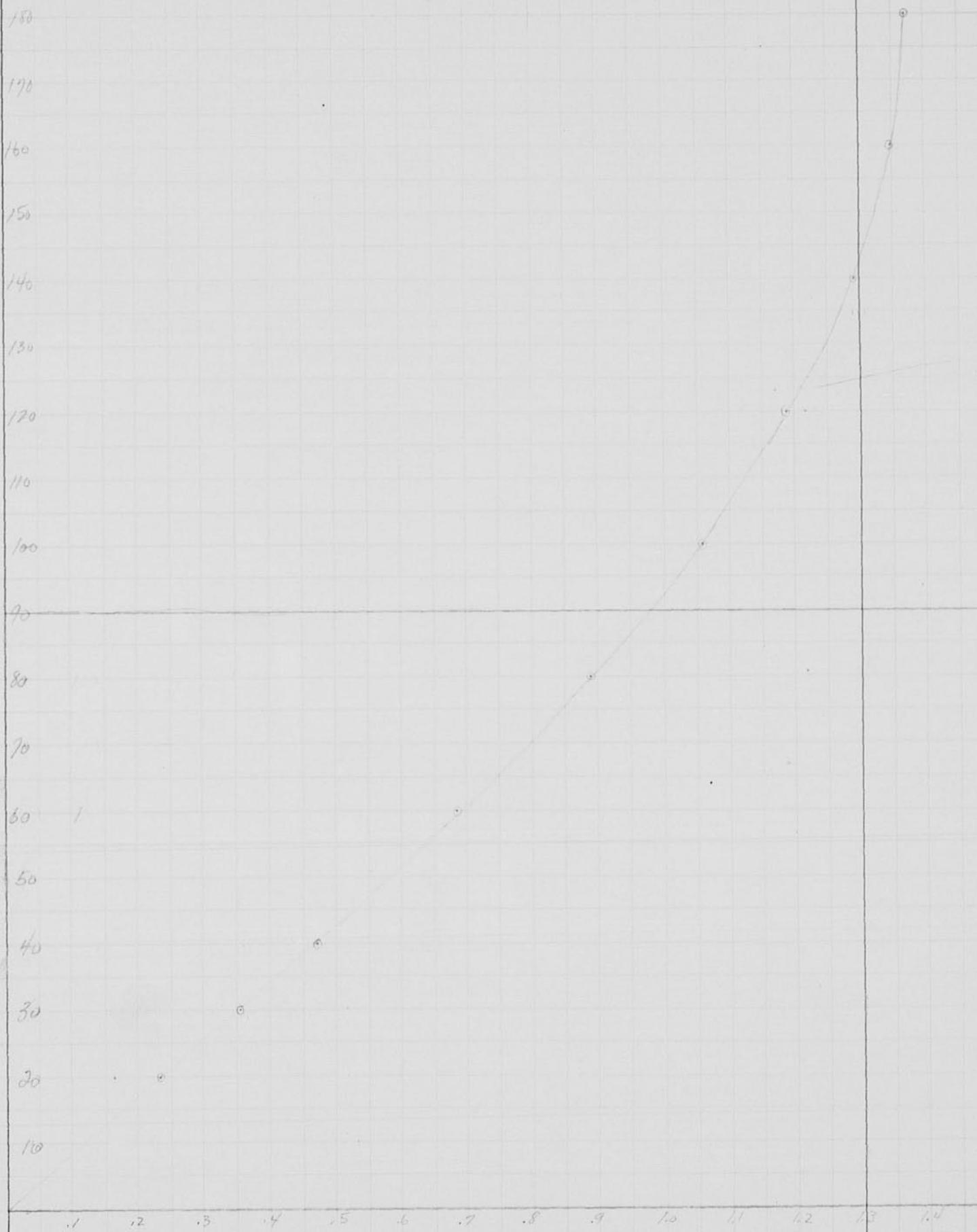
Notebook # 3

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Calculation of Oscillogram No. 9.

May 4 1920
H. T. Edgerton

No.	angle Volts mch.	ϕ deg	P mch.	P kw.
1	1.32	+147	.20	1.53
2	1.22	124	.35	2.69
3	1.1	107	.55	4.22
4	.97	88	.66	5.07
5	.82	72	.53	4.07
6	.61	52	.21	1.61
7	.41	35	-.03	-.23
8	.22	18	-.16	-1.23
9	.00	00	-.12	-0.922
10	.24	21	+1.03	+2.3
11	.42	36	.18	1.38
12	.60	51	.33	2.54
13	.79	69	.54	4.15
14	.97	88	.69	5.30
15	1.10	107	.57	4.38
16	1.20	122	.32	2.46
17	1.29	140	.05	.384
18	1.35	161	-.13	-1.0
19	1.37	180	-.15	-1.15
20	1.35	161	-.02	-.154
21	1.32	147	+.16	+1.23
22	1.25	130	.30	2.3
23	1.15	114	.53	3.84
24	1.02	94	.66	5.07
25	.85	75	.58	4.45

Calc of slip.

180 degrees = 15 cycles of time

$$\frac{d\theta}{dt} = \frac{180}{.25} = 720 \text{ elec deg/sec.}$$

$$\omega = 360 \times 60 = \text{sync speed in elec deg/sec} = 21,600 \text{ elec deg/sec.}$$

$$s = \frac{d\theta}{dt} = \frac{720}{21,600} = 3.33 \text{ percent slip.}$$

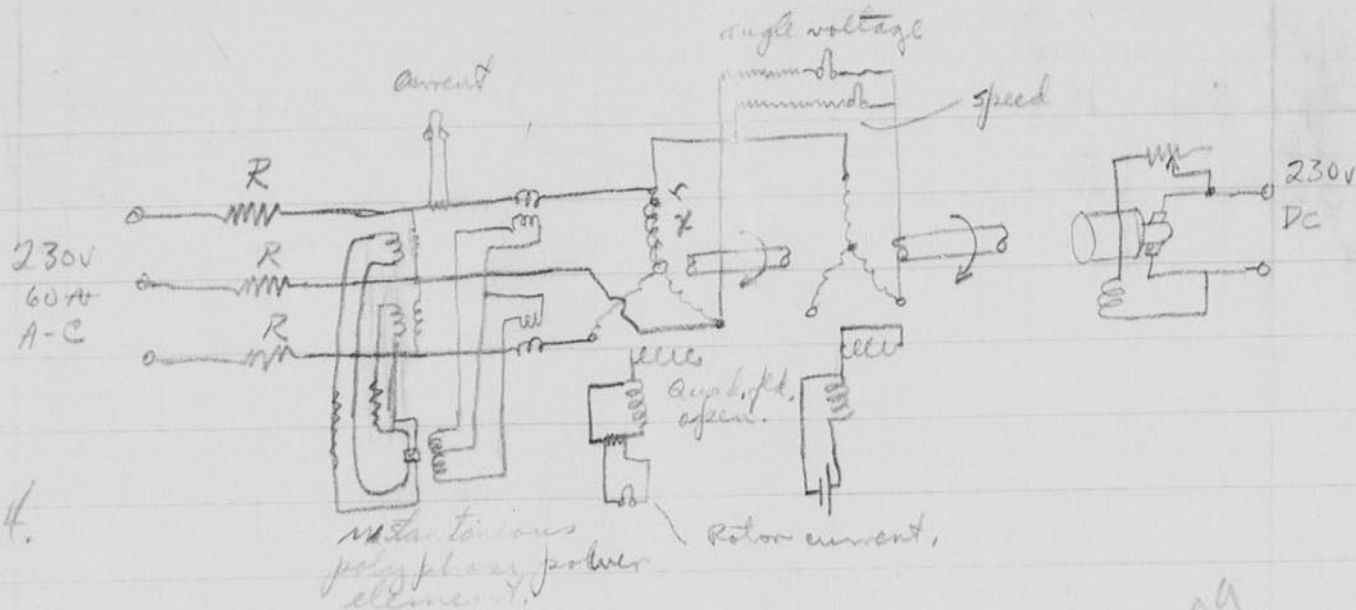


Fig 4.

wiring diagram for oscillogram no ¹⁸⁹ X?

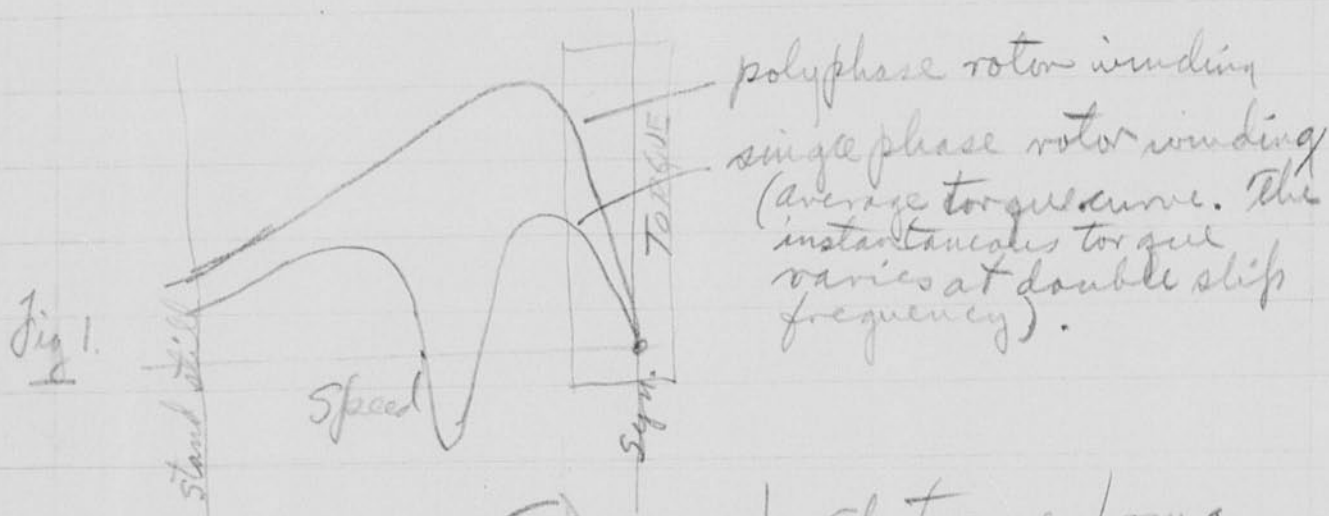


Fig 1.

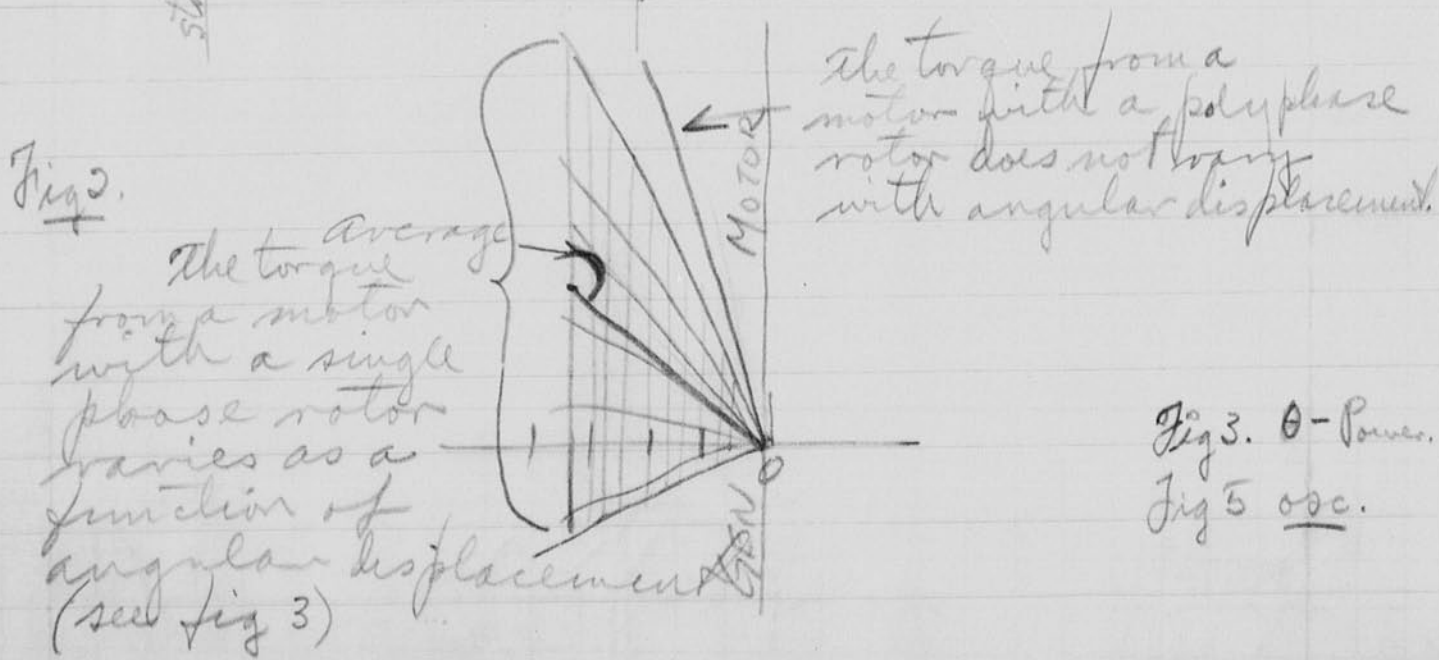


Fig 2.

Fig 3. θ -Power.
Fig 5 osc.

Notebook # 3

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___ negative strip(s)

3 unmounted page(s)
(notes, drawings, letters, etc.)

was/were filmed where originally located between page 28 and 29.

Item(s) now housed in accompanying folder.



E T_{max}
 O T_{min}

OSC. No. 11
 PAGE 3-26
 MAY. 4, 1930
 H. E. EBBERTON



Osc. No. 11.
 Page 3-26
 MAY 4, 1930
 H.F. EDGERTON

Self-Oscillation of 96.

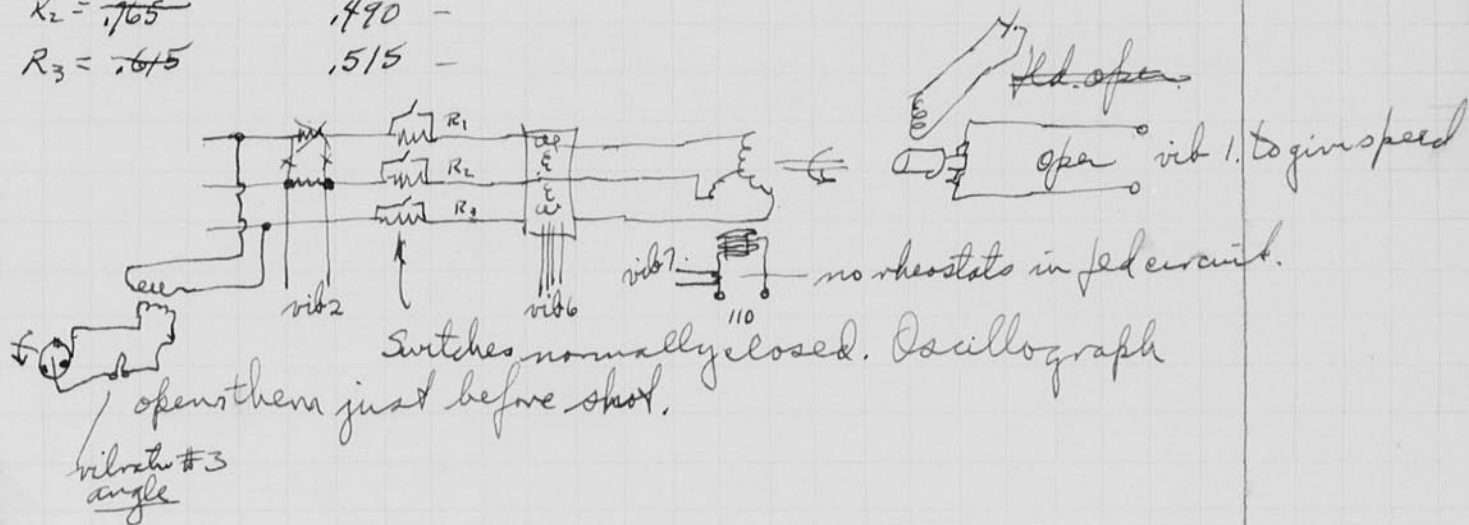
May 6 1930
 H. J. Edgerton
 John Ross.

Salient pole syn. motor # 305997.
 G.E. Co 1TB-6-37.5-1200 Form PB
 PF-80 3φ 90-180 amps 30 kW. 60 Hz. ~~37.5~~
 Direct Connected to.

1016477 shunt type RC B3 A Jwm
 230 volts amps 160 1200 R.P.M..

lead resistances.
 $R_1 = .507 - .128$.466 - .128
 $R_2 = .765$.490 -
 $R_3 = .615$.515 -

Osc.
 15



16
 17
 18
 19.

Calibration			W	I_f	Dc motor		
I	V	x 20			I_f	E	I
50.3	221	246		282	2.22	227	69.0
		.981					

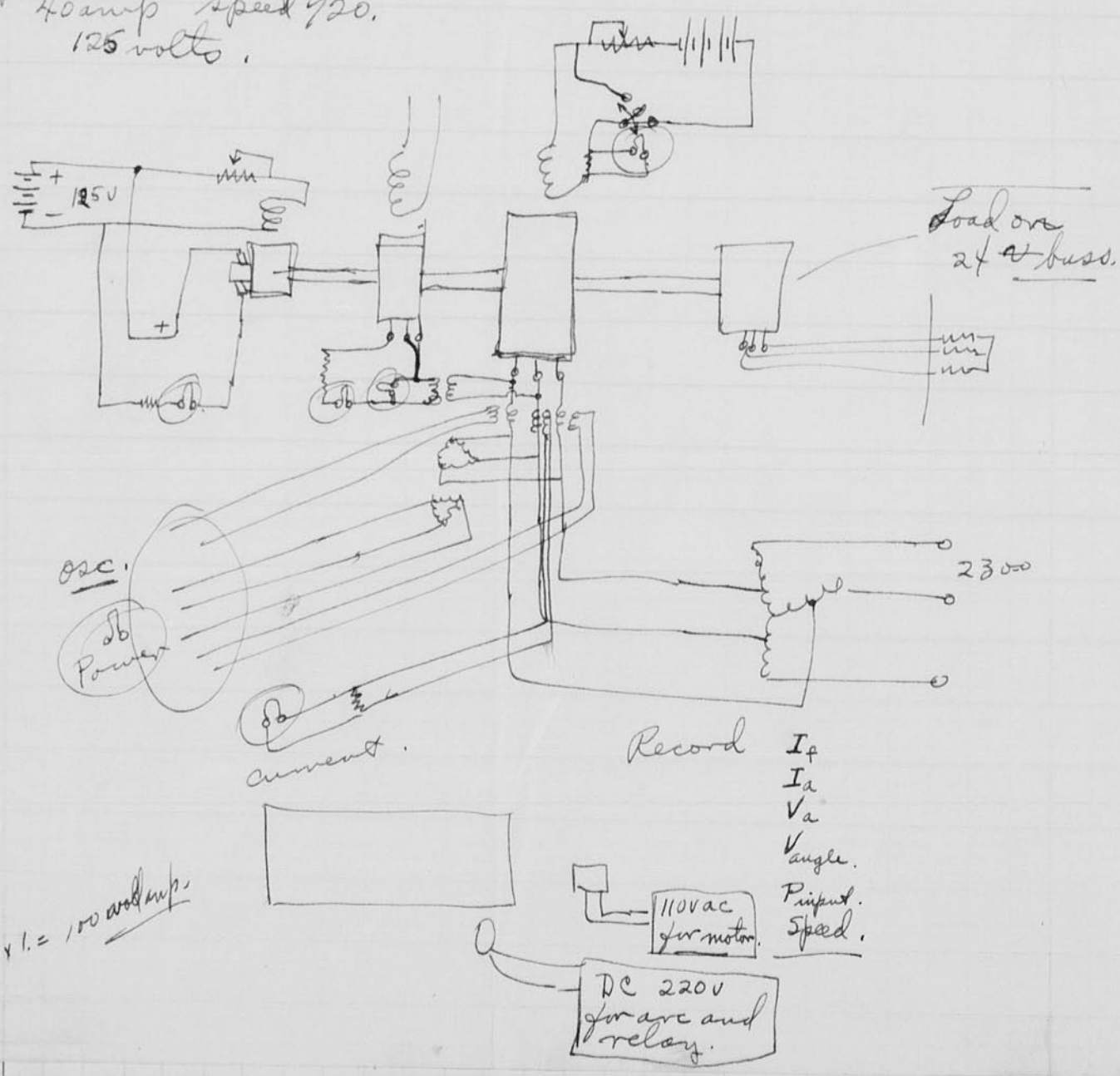
I_f on 230 V.

Salient-Pole Motor Outline of Pullin Tests

May 12, 1930
H. E. Edgerton

<p>S. E. A.C. Generator # 842053 ATI-10-62.5M 720 C 50 KW 720 p.f. 8 157 amp volts 230</p>	<p>S. E. Synchronous Motor # 302707 ATI-10-160M 720 40 amp 2300V 60Hz H.P. output 160 speed 720 P.f. = .8 <u>125 KVA.</u></p>	<p>S. E. A.C. Generator # 842052 ATI-4-62.5-M-720 C 50 KW speed 720 p.f. = 8. amp 157. Volts 230.</p>
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S. E.
D.C. Exciter
502320
type FF-4-5-720 form E
40 amp speed 720.
125 volts.



May. 14, 1930
H. Edgerton

WR^2 from Ira A. Terry's letter of May 5, 1930.
and reactances.

#	WR^2	X_d	X_q
# 842053	960	.362	0.173 .173.
# 302707	1175	.651	.32
# 842052	627	0.409	0.209
Total WR^2		2762 pound feet squared.	

$$P_j = \frac{18.6 WR^2 f}{p^2} \times 10^{-6} = \frac{18.6 \cdot 2762 \cdot 60}{10 \times 10} \times 10^{-6} = 0.0308 \text{ kw/elec deg/sec}^2$$

Reluctance power at $\frac{1}{2}$ voltage.

$$V = .5$$

$$X_d = .651 \quad X_q = .32$$

$$P = \frac{V^2 (X_d - X_q)}{2 X_d X_q} = \frac{.5^2 (.651 - .32)}{2 \cdot .651 \cdot .32} = 0.198$$

$$P_{kw} = \frac{125}{160} \cdot 0.198 = \frac{32}{160} \cdot 0.198 = \underline{8. KW.} \quad \underline{25. KW} \quad 31.8 KW.$$

Synchronous power.

$$P_m = \left(\frac{VE X_d}{X_d X_q} \right) \frac{125}{160} = \left(\frac{.5 \cdot .5}{.651} \right) \frac{125}{160} = \underline{31.3 KW.} \quad \underline{61.3 KW} \quad \underline{48.0 KW}$$

1100 volts - 8.8 amperes field current. from sat. curve.

$$\frac{31.3 KW \times 1000}{\sqrt{3} \cdot 1100} = \underline{16.4 \text{ amperes.}} \text{ required from a.c. 1100 volt source.}$$

The motor requires 160 amperes exciting current.

May 15, 1930
 F. E. Dyer
 Fritz Broderick

Test on 302707.

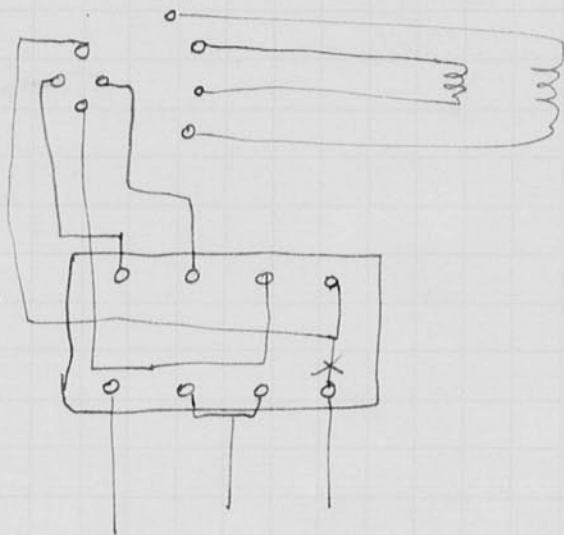
40:1
 C.T. 200:1 ratio
 P.T. 20:1.

V. ¹²⁰⁰ 55 on start III - 115+ start.
 58 on run.

I less than 1 amp run.
 3 amps on start ⁴⁰ ¹²⁰

32. = 1.6 amp x ²⁰ ~~100~~ running light.
 32. amps.

Connections of polyphase watt element of oscillographs



Field Resistance.

9.5 ohms fld + Rheo on board.

5.2 ohms fld only with Rheo out.

$$s \leq \frac{600 \pm}{n} \sqrt{\frac{P_m}{f(WR^2)}} \quad n \text{ syn speed.}$$

$P_m = 50$
 $f = 60$
 $WR^2 = 2762$
 $n = 720$

$$s < \frac{6}{720} \sqrt{\frac{50}{60 \cdot 2762}} < 1.45\%$$

61.5
 see page 32
 for connected.

Steady-State Power-Angle Curves on #302707.

May 16 1980.

H.E. Edgerton
Paul Fournier
Sam Levine
Chris Kingsley

θ	V_1 a. Chae Gen	V_2 Gen	Wage. 32270.	V_1 x20 24873.	HW x10 x20	I_1 x10	I_2 x10	I_f D.	Prot. KW meter on board.	V
	116.5	116.5	89.0	115.3	1.7 34.	1.08	1.05	18.9	24	130.
	56.5		115.0	2.28 45.6	1.34	1.83	18.8	34	155	
	56.0		115	2.44 48.8	1.43	1.42	18.95	38	164	
	53.9		115.3	2.85 57.0	1.61	1.6	18.8	46.	180	
	61.0		115.2	1.37 27.4	1.-	.95-	18.8	18.	112.	
	64.3		115.2	.90 18.0	.75	.75±	18.8	8.0	70	
	66.0		115.2	.40 8.0	.65	.65	18.9	0	0	
	66.5		115.2	.5 47	-	-	18.9	0	220	
	52.		115.0	3.46 69.2	1.86	1.88	18.9	57	230.	
	50.		115.0	3.92 78.4	2.1	2.1	18.9	66	250.	
	66.5		115.2	.5	3.46	3.46	9.2	0	0	
	64.0		115.0	1.18	3.5	3.51	9.25	9.0	97	
shut down	60.0		115.3	1.61	3.55	3.56	9.25	20.4	138	
	56.		115.0	2.06	3.71	3.70	9.20	28.5	166.	
	54.5		115.0	2.30	3.77	3.74	9.20	33.2	176	
	52.5		115.0	2.64	3.84	3.82	9.20	39.5	192.	
	49.5		115.0	3.17	3.97	3.95	9.20	49.0	213	
	47.5		115.0	3.47	4.07	4.07	9.2	55.0	227	
meters interchanged	45.5		116.0	3.82	4.18	4.20.	9.1	60.	239	
	43.0		115.8	4.17	4.33	4.33.	9.1	66.5	252	



1/2 volt on autotrans (3.2?) guess from yesterday tests

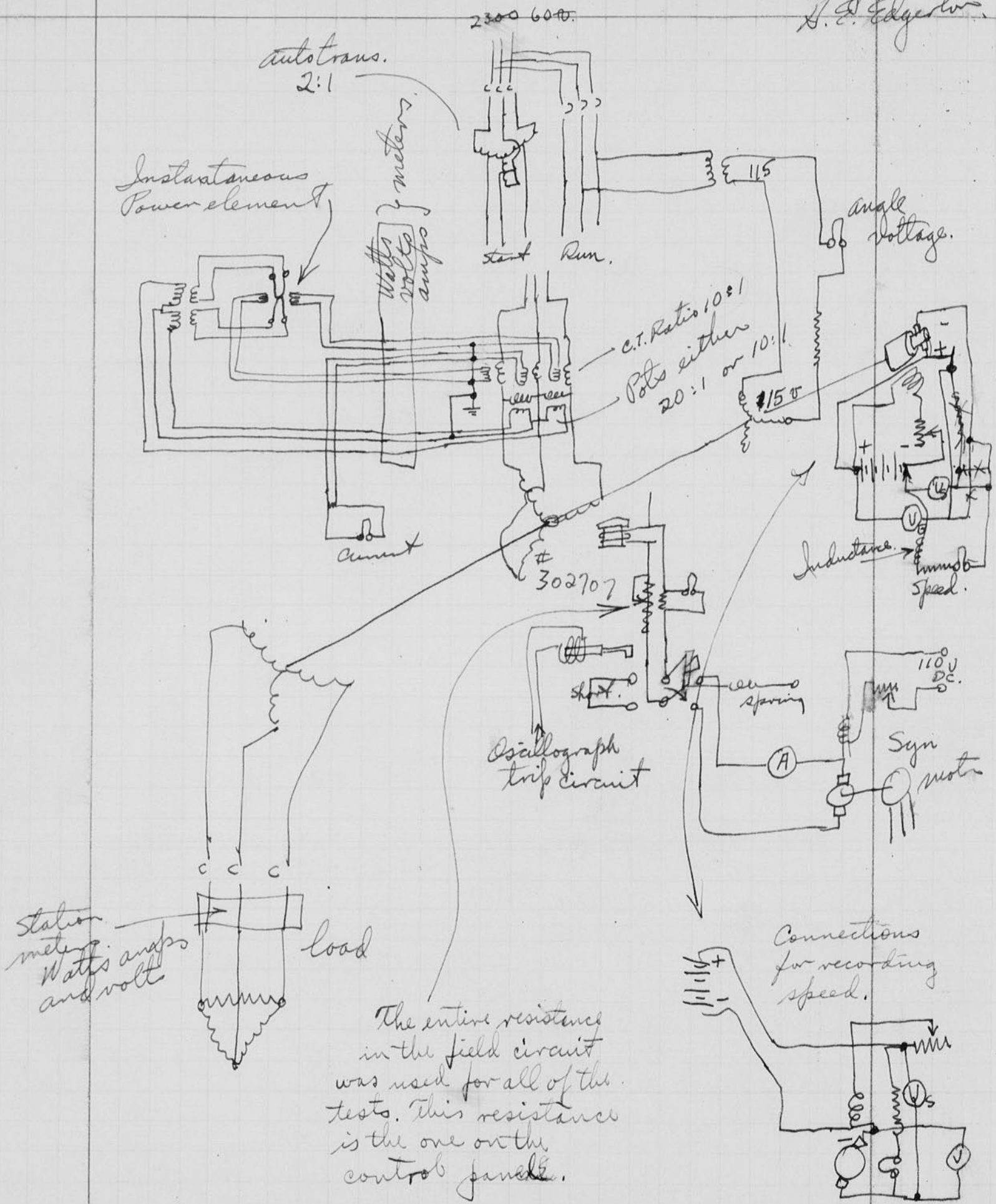
shorted thru 5 ohms

32. } Reluctance pullout
↓
16. } Pullin.

2 AM $\frac{.285}{233.0} = 16.54 \times 10^{-2} = 33.07^\circ$

core loss of 24 W

May 17 1930
H. P. Edgerton



and Slip Torque Curve of 302707.

H.S. Edgerton
May 17, 1936

V angle
mechanism

V x10	I x10	HW x10x10	I _f	V _B	V _s max min.	P _{out}	V _{out}
115	3.03 5.+	.80 3.- 4.7	0		.22 .33 .1 - .3	0 25	175
114	5.+	1.3-4.0				18	148

Kingley
Paul Toumairie
Lennig

Osc	V	I	HW	I _f	V _B	V _s	P _{out}	V _{out}
20		1.71	3.4	9.3			25.5	175
21	115	1.69	3.3	9.4			25	173
22	115	1.82	3.61	9.4			27.5	182
23	115	1.84	3.61	9.2			27.5	157
24		Off scale - failed to pull in					27.5	155
25	114.5	1.84	3.7	9.2	19.8		27.3	152
26	115	1.86	3.69	9.2	19.8		27.8	154
27	115	1.84	3.67	9.25	19.8		27.7	153
28	115	1.86	3.72	9.55	19.8		28.2	155
29	115	1.86	3.66	9.4	19.8		27.8	153
30	115	2.11	3.98	8.55	19.5	24.5 angle speed	31.0	161 calibration

0 ohms in series with V_s
2 ohms in series with V_s

These readings were made immediately after synchronization.

Slip of film 30 $\frac{100 - 0.2}{19.5} = 1.025\% = \frac{(1.07 - .80)}{2}$ inches variation .94 in.

Calculation of slip from number of cycles per 360 elect degrees
x cycles/elect rev.

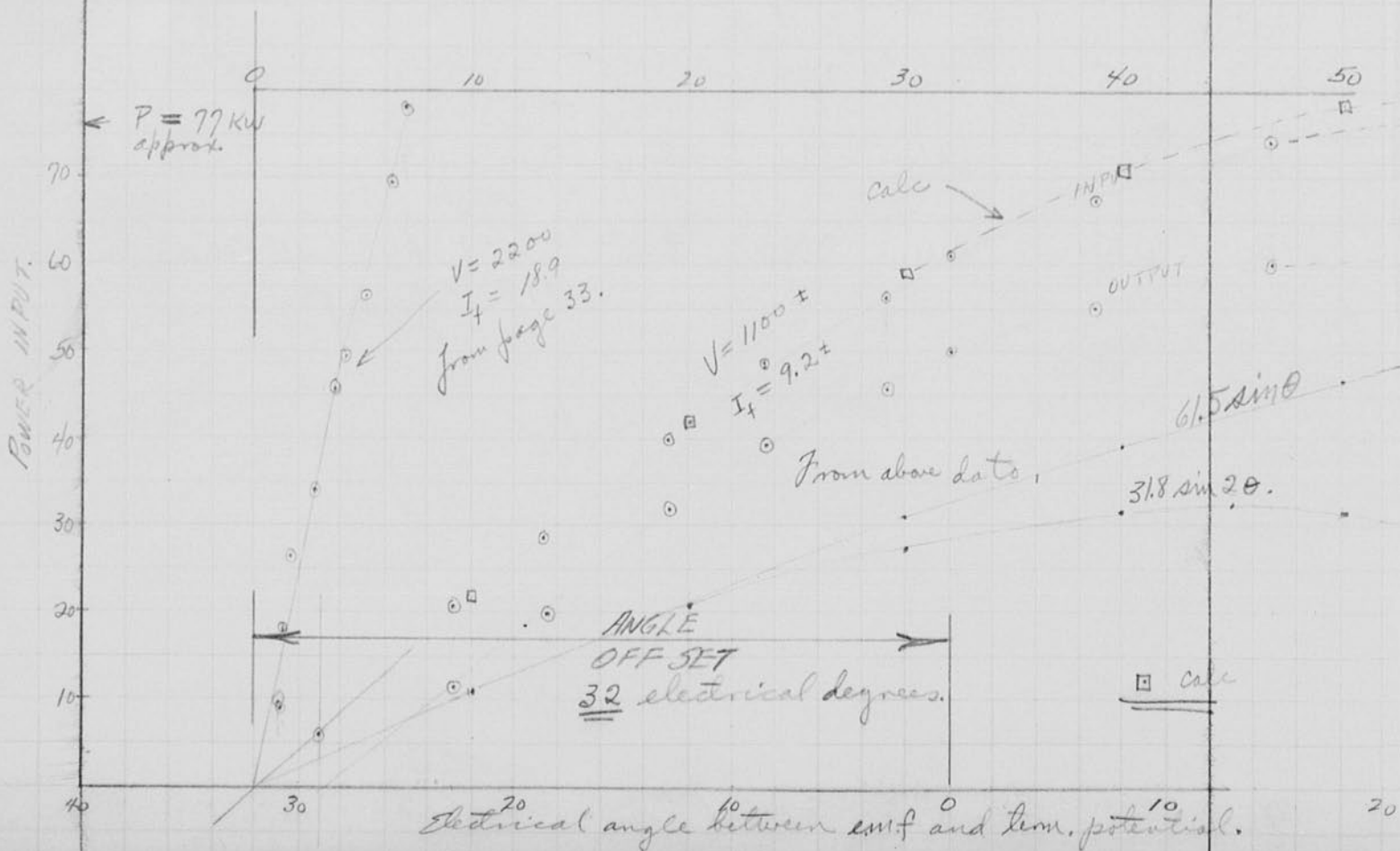
slip = $\frac{60}{x}$ cycles/sec % slip = $\frac{60 \times 100}{60x} = \frac{1}{x} \times 100$

Steady-state

May 17, 1930.

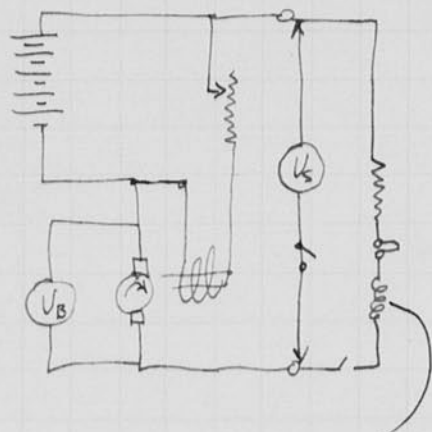
Power - Angle Curve 302707

V_{ac} $\times 10$	i_{ac} $\times 10$	i_{ac} $\times 10$	kW $\times 10 \times 10$	$V_{ang.}$	P_{out}	V_{out}	i_f	$\frac{V_{ang.}}{2 \times 10^{16.5}}$	θ	θ
115.2	-	-	0.61	59.	0	0	9.2	.253	14.55	29.1
115.5	1.05	1.02	2.05	46	12	105	9.1	.197	11.35	22.7
115.1	1.46	1.44	2.82	38	20	132	9.1	.163	9.39	18.78
114.9	2.03	2.05	4.00	26	32	161	9.2	.111	6.4	12.8
114.6	2.53	2.56	4.87	17	39	181	9.2	.077	4.78	9.36
115.0	3.04	3.02	5.65	6 [±]	46	196	9.2	.056	3.45	6.90
114.2	3.33	3.32	6.15	0 [±]	50	205	9.15	0	0	0
114.3	3.74	3.75	6.72	13	55	215	9.2	.055	3.2	6.4
114.2	4.40	4.42	7.40	30	60	225	9.2	.029	1.73	3.47



Calibration of speed vibrator

May 19, 1930.
H. E. Edgerton
Paul Bourmarier



V_B	V_S	Def in inches.	slip	slip/inch
19.4	.20	.96		
19.4	.30	1.31		
19.4	.30	1.22		
19.4	.20	.80	after the commutator was cleaned.	
	.32	2.03		

(220 volts
.3 amps ±)
60 cycles.
Less than .5 of an amp.

these show that the speed cannot be calibrated by this means.

With $\frac{1}{3}$ of inductance 220 v 3.0 amps 60 Hz.

Average slip may be computed from the angle voltage. Then this should give the average of the speed record. It also may be computed from the record of field current since the fundamental of this is at slip frequency.

There was too much inductance in series with the speed vibrator for the tests taken on the 17th. However the records show the tendency of the speed which is important and sufficient for us.

Synchronization Tests # 302707

May 14, 1930
 J. E. Edgerton
 Paul Dougherty
 O = did not sign.

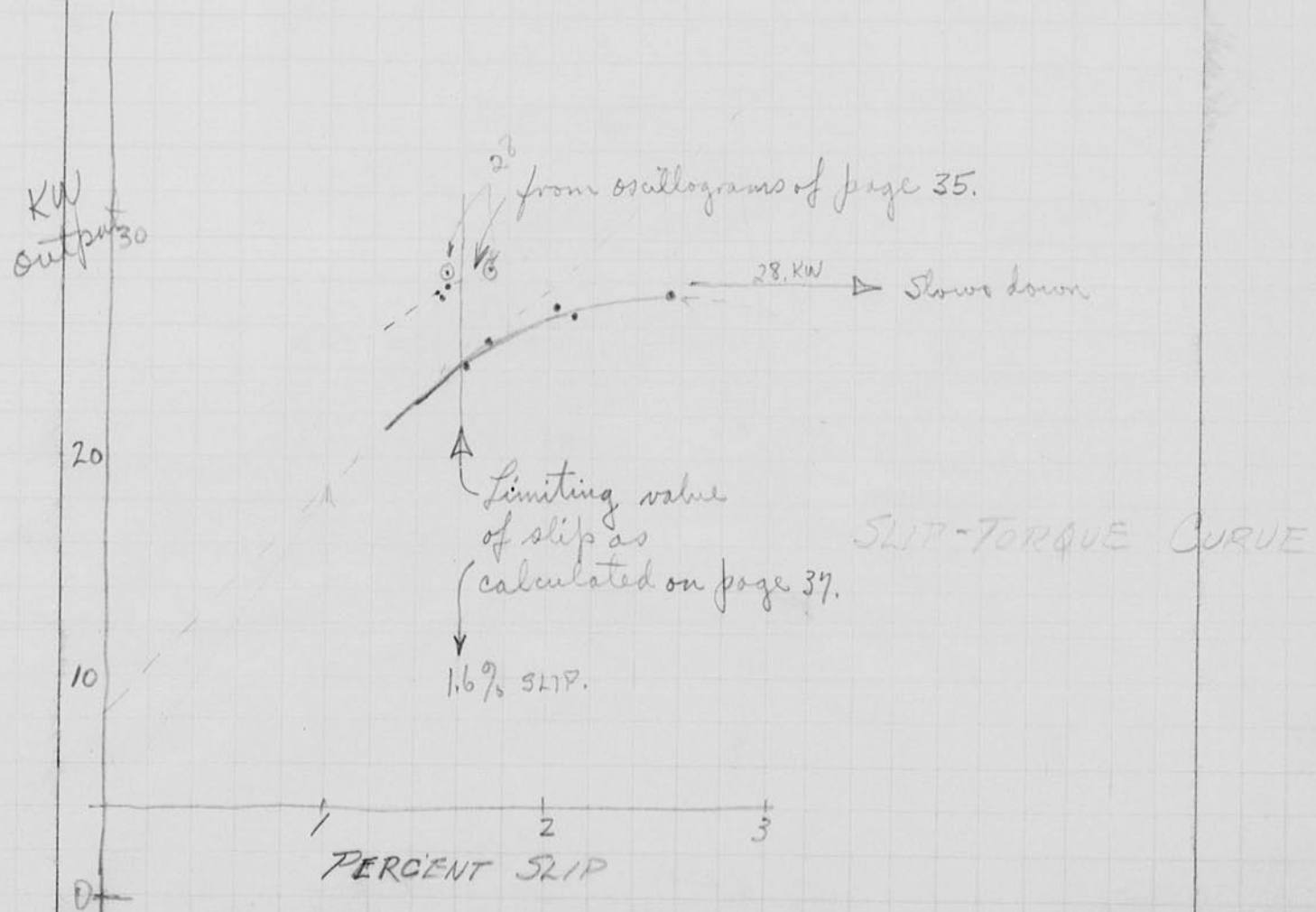
V I_f $\frac{H \cdot W}{\text{output}}$ I_a P V. $x = \text{Syn.}$
 $\times 10$ \checkmark $\times 10 \times 10$ $\times 10$ output out.

9.2- 28.-
 A load of 28 KW causes the slip to be much larger than it was yesterday Sat.
 slip %

29 beats in 1/2 min.	.61	24.1	158
113.6 31 "	1.724	25.3	164.5
113.6 38 ..	2.11	26.5	171.0
113.5 46	2.56	27.6	176.0
37 $\frac{.32 \text{ volts}}{19.4}$	1.65 2.05	26.5	170.0

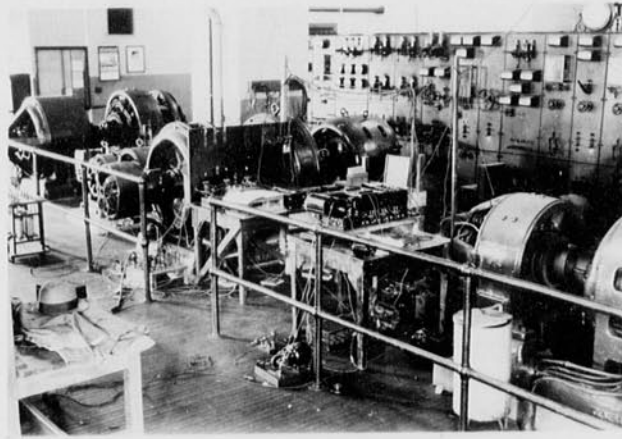
$\% \text{ slip} = \frac{\text{beats}}{36.00} \%$

with fld shorted the motor pulls out at 26.5 kw.

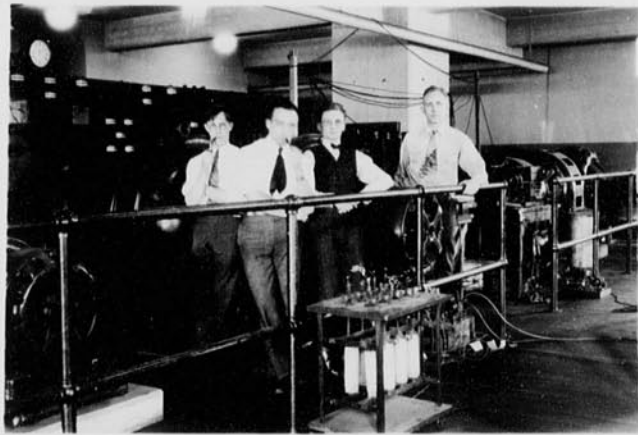


Photographs taken May 17, 1930
during tests of pages 34-35-

May 22, 1930
H. E. Edgerton

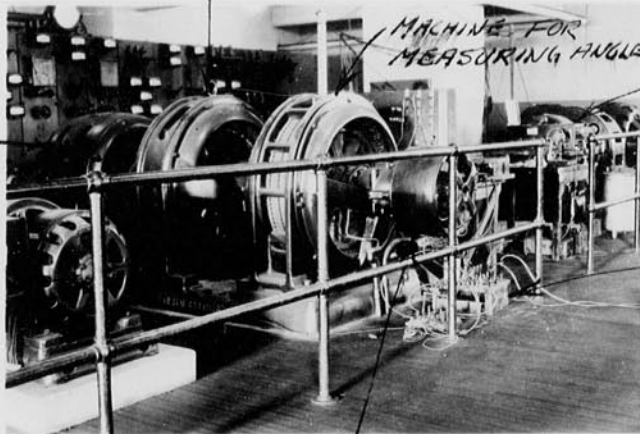


Chas Knigzaley
Sam Levine
Paul Toumanian
Harold Edgerton



302707
160 KW MOTOR BEING TESTED.

50 KW
LOAD
MACHINE



MACHINE FOR
MEASURING ANGLE

oscillograph

change-over
switch.

Exciter used to
measure slip.

Notebook # 3

Filming and Separation Record

___ unmounted photograph(s)

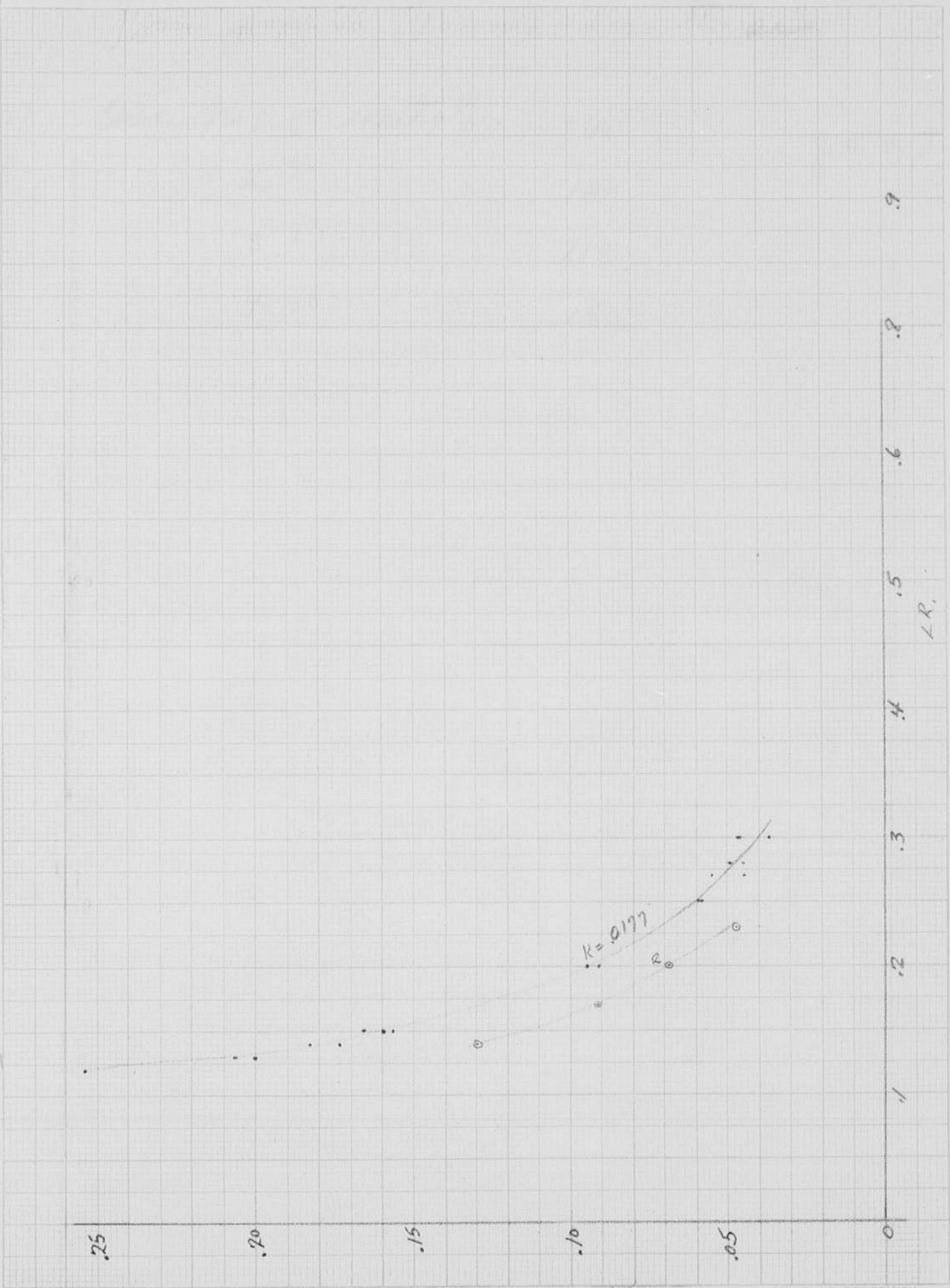
___ negative strip(s)

1 unmounted page(s)
(notes, drawings, letters, etc.)

• was/were filmed where originally located between page 40 and 41.

Item(s) now housed in accompanying folder.

8/0



From page 68 Journaier's thesis.

Osc No 1. 27 excitator -40°

2 24 130°

3 23 110°

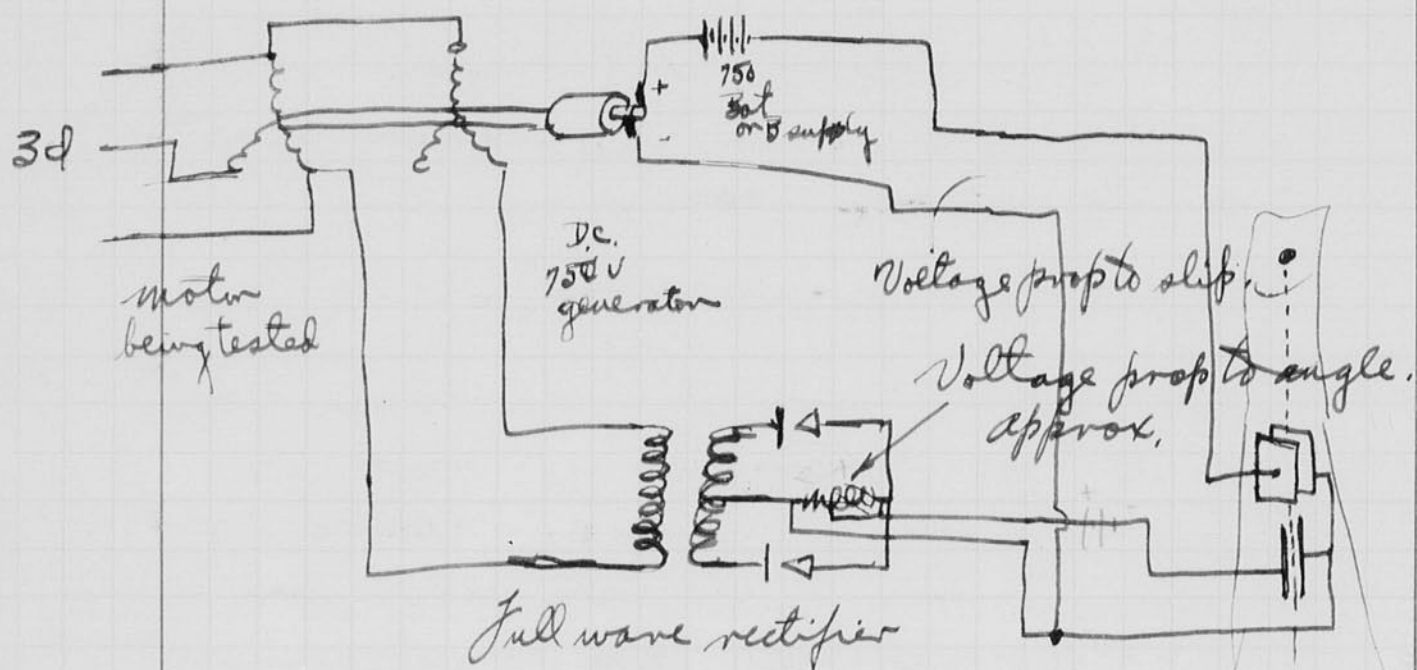
4 28 $150.$

$$g = 0.7$$

Oct 22, 1930
J. E. Edgerton

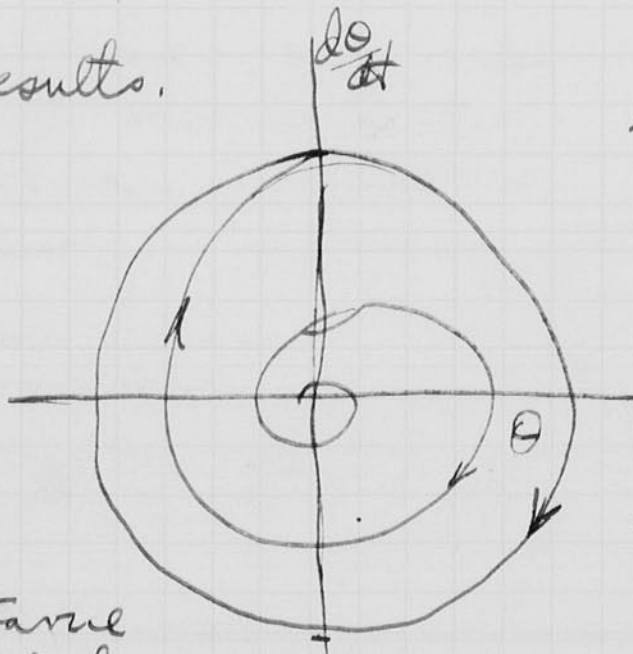
Method of measuring Slip-angle curves for a hunting motor.

A W. E. Braun tube can be used to give a plot of slip against angle by the method shown.



Expected results.

The spot will give an increasing spiral if the damping is negative. If the damping is positive the any disturbance will give a spiral that decreases to a point at the center.



Determination of the Moment of
Inertia of a Synchronous Machine by
Measuring its Hunting Period.

The motional differential equation of synchronous torques that determines the angular transients of a synchronous machine that is connected to an infinite bus is the following:

$$P_j \frac{d^2\theta}{dt^2} + P_m \sin \theta = \text{load (any function of time)}$$

The electrical damping term has been neglected since it is usually small. The first term represents the torque that is due to the acceleration or deceleration of the combined rotating mass of the motor and load. The second represents the synchronizing torque between the two components of magnetic field which are due to the applied armature voltage and the field current.

The coefficients P_j and P_m will now be determined in units of kilowatts, electrical degrees, and seconds.

$$\text{Inertia torque} = J\alpha \quad \text{in pound feet}$$

$$\text{where } J = \frac{WR^2}{g} = \text{the moment of inertia in poundals}$$

$$\alpha = \text{the angular acceleration in mechanical radians per second}^2.$$

$$\text{Inertia power} = \frac{2\pi}{60} (J\alpha) = \frac{2\pi}{60} \left(\frac{WR^2}{g} \right) \alpha \quad \text{in pound feet per second.}$$

If we express the acceleration in electrical degrees per second² then

$$\alpha = \frac{2\pi}{180p} \frac{d^2\theta}{dt^2}$$

since 2π mechanical radians = $180p$ electrical degrees.

-2-

The expression for the inertia power can be now written in terms of kilowatts, electrical degrees, and seconds as

$$\begin{aligned} \text{Inertia power in kilowatts} &= \frac{2\pi}{60} \left(\frac{WR^2}{g} \right) \left(\frac{2\pi}{180p} \right) \frac{d^2\theta}{dt^2} - \frac{.746}{550} \\ &= P_j \frac{d^2\theta}{dt^2} \end{aligned} \quad (2)$$

$$\text{where } P_j = 0.154 \times 10^{-6} \frac{n(WR^2)}{p} \quad \text{or } 15.4 \times 10^{-6} \frac{f}{p^2} (WR^2)$$

$$\text{since } n = f \frac{120}{p} \quad \text{r.p.m.}$$

The maximum synchronizing power for a three-phase round-rotor synchronous machine which has negligible armature resistance is

$$P_m = \frac{3 VE}{1000 x} \quad \text{kilowatts} \quad (3)$$

where V = terminal phase voltage

E = induced phase voltage due to the field current.

x = synchronous reactance per phase.

If the magnitude of the angular oscillation is not large then the slope of the power angle curve may be considered as a straight line. This assumption makes equation 1 a linear differential equation. The slope of the power-angle curve is

$$\text{Slope} = P_m \frac{\pi}{2} \frac{\cos \theta}{90} \quad \text{kilowatts per electrical degree}$$

and the linear differential equation is

$$P_j \frac{d^2\theta}{dt^2} + P_m \frac{\pi}{2} \frac{\cos \theta}{90} \theta = \text{load in kilowatts.}$$

Notebook # 3

Filming and Separation Record

___ unmounted photograph(s)

___ negative strip(s)

1 unmounted page(s)
(notes, drawings, letters, etc.)

was/were filmed where originally located between page 44 and 45.

Item(s) now housed in accompanying folder.

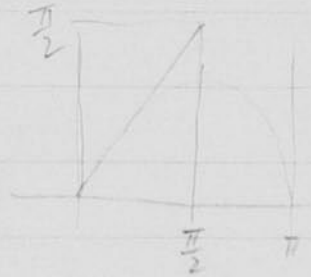
$$P_R = \frac{P}{\theta}$$

$$f = \frac{120}{\theta}$$

$$n = \frac{120f}{\theta}$$

$$F = 0.268 \sqrt{\left(\frac{VE \cos \theta}{x}\right) \frac{P^2 (1)}{f (WR^2)} \frac{(120)^2 f}{(120)^2 f}}$$

$$= 0.268 \sqrt{\frac{VE \cos \theta (120)^2 f}{x n^2}}$$



$$= \frac{0.268 \times 120}{n}$$

$$= \frac{1}{2\pi} \sqrt{\frac{1}{18.5 \times 10^{-6} \frac{f}{P^2} WR^2} \cdot \frac{P_m \frac{\pi}{2} \cos \theta}{\frac{\pi}{2}} \frac{\frac{\pi}{2}}{90}}$$

$$= \frac{1}{2\pi} \sqrt{\frac{1}{18.5 \times 10^{-6}}} \sqrt{\frac{P^2}{f (WR^2)} \frac{f (120)^2}{f (120)^2}} P_R$$

$$= \frac{120 \times 10^3}{2\pi} \sqrt{\frac{1}{18.5}} \frac{1}{n} \sqrt{\frac{f}{WR^2}} P_R$$

$$= \frac{.054}{0.233} \quad 2400$$

$$\sqrt{\frac{\pi}{180}} \frac{120 \times 10^3}{6.28} \times .233 =$$

4430. cyc per second
 588.0 cyc per second
 3540 cyc per min

10174
 112/ 3.00 .132

-5-

The differential equation is analogous to a series circuit of inductance L and capacity C which has the differential equation

$$L \frac{d^2 q}{dt^2} + \frac{q}{c} = E.$$

The natural frequency of oscillation of an electrical circuit is known to be

$$F = \frac{1}{2\pi} \sqrt{\frac{1}{Lc}} \quad \text{cycles per second.}$$

Similarly the frequency of mechanical oscillation is

$$F = \frac{1}{2\pi} \sqrt{\frac{1}{P_j} \frac{P_m \sum \cos \theta}{90}}$$

$$= 0.268 \sqrt{\frac{VE p^2 \cos \theta}{x f (WR^2)}}$$

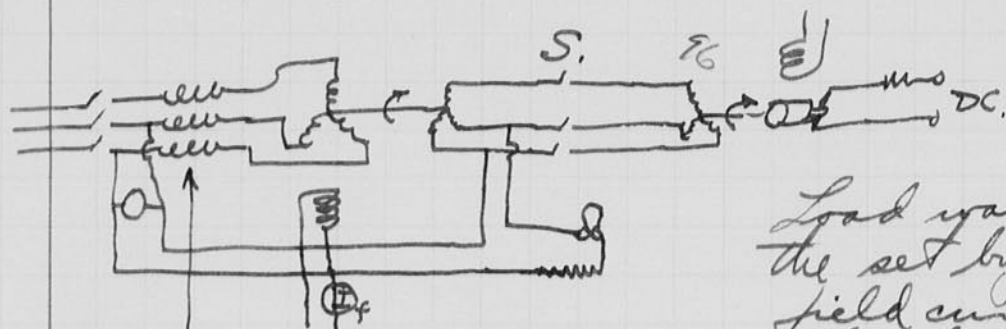
Solving for the moment of inertia

$$WR^2 = 0.072 \frac{VE p^2 \cos \theta}{x f F^2}$$

H. E. Edgerton
M.I.T. Nov. 15, 1930

H. S. Edgerton.
Nov. 16, 1930.

WR² from Hunting tests.



Reactors series connection
good for 30 amperes.

Load was put on the set by changing field current on the d.c. machine. Then switch S was opened and this sudden removal of load allowed the set to swing about its no load angle. The quadrature field was open circuited and so the damping was negligible.

An oscillogram was taken when the circuit conditions were as follows.

$V = 232$ volts line to line.

$I_f = 8.4$ amperes.

$I_{ac} = 5 \pm$ amps.

Calculation of WR^2 from data of preceding page.

$$V = \frac{232}{\sqrt{3}}$$

$$E =$$

$$p = 6 \text{ (number of poles)}$$

$$\cos\theta = 1$$

$$x =$$

$$f = 60 \text{ cycles/sec.}$$

$$F = \text{cycles/sec.}$$

$$0.072 \frac{VE p^2 \cos\theta}{x f F^2} =$$

see next page.

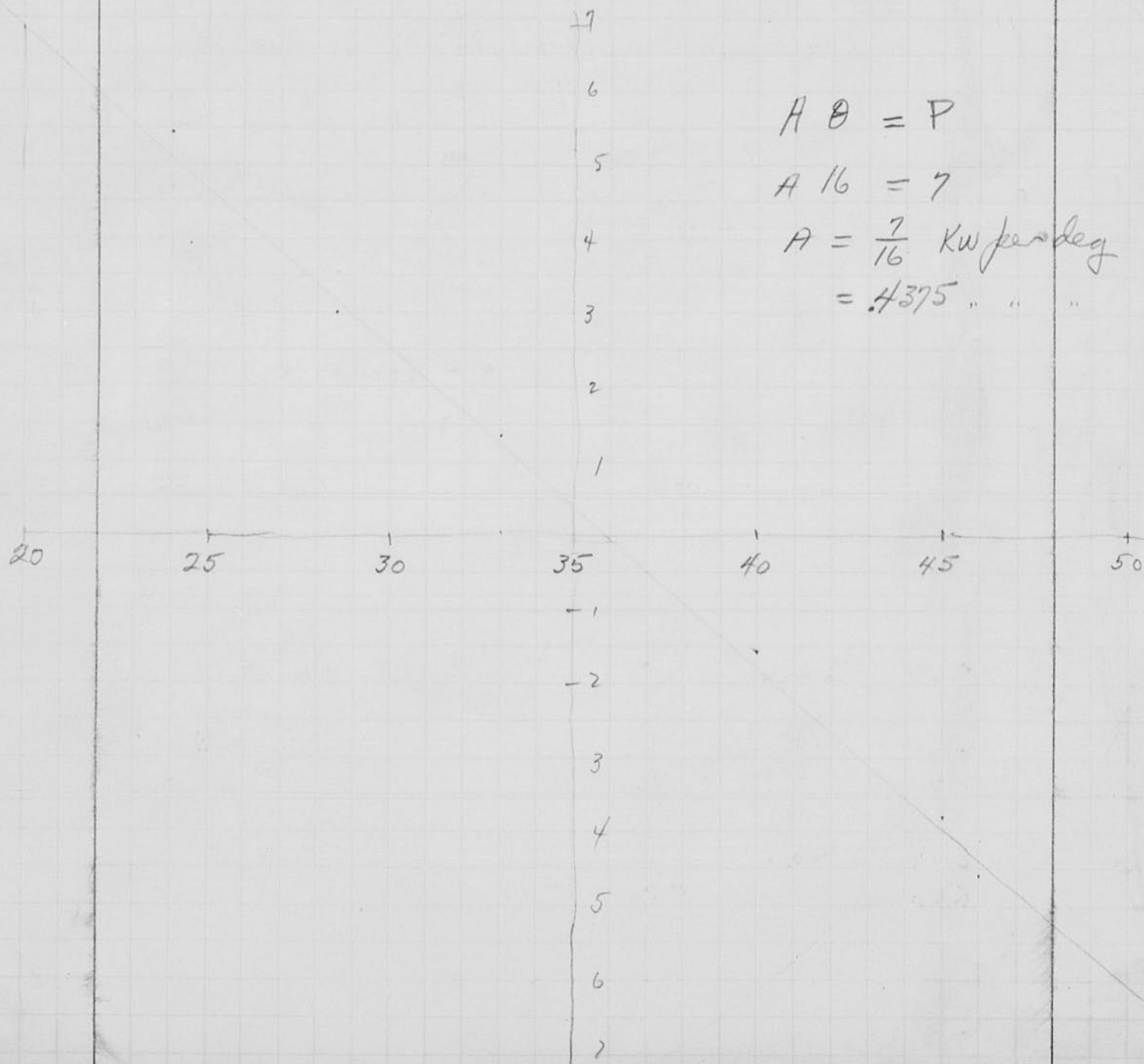
2 KVA

804 B. Syn Generator.

Starting Condition

60 cycles	V_{stator}	18	17	16	23.5	28.0	27.0
	V_{field}	186 volts			285		

When cold this motor will not start with 182 volts



Power-Angle Curve of 804 A.

Nov 17 1930

θ	I_1	I_2	I_3	KW ₁	KW ₂	V	I_f	E_{KW}	Reactors in series 804A.
+40.0	4.2	-	3.75	$\times 2$.37M	$\times 2$.43M	227	8.15	-1.60	
+45.0	9.8	9.5	9.0	$\times 2$.94M	$\times 2$.95M	227	8.15+	-3.78	
+33.0	3.25	-	3.65	$\times 2$.294	$\times 2$.394	226.5	8.10	+1.36	
28.5	7.50	7.5	7.8	$\times 2$.684	$\times 2$.824			+3.0	$\frac{232.84}{226.81}$
24.0	-	12.8	13.0	$\times 4$.5654	$\times 4$.714	228	8.05+	+5.4	
19.0 18.0	-	17.6	17.9	$\times 4$.744	$\times 4$.9854	226	8.0	+6.9	
46.0	-	11.8	11.0	$\times 4$.59M	$\times 4$.56M	225.5	8.05	-4.6	
53.0	-	19.2	18.3	$\times 4$.98M	$\times 4$.875M	226	8.05	-7.42	

$$F = \frac{50}{60}$$

Short circuit of 804A + reactors
as used in tests.

I_f	I_a	E
2.05	16.2	
3.55	28.2	
3.60	28.2	109
4.95	38.9	
4.99		150
6.16	48.2	
6.16		182

$$F = \frac{1}{2\pi} \sqrt{\frac{1}{P_j} 0.4375}$$

$$P_j = \left(\frac{1}{2\pi}\right)^2 \frac{1}{F^2} 0.4375$$

$$WR^2 = \left(\frac{1}{2\pi}\right)^2 \frac{1}{F^2} 0.4375 \frac{P^2 \times 10^6}{18.6 f}$$

$$WR^2 = \left(\frac{50}{2\pi \times 60}\right)^2 0.4375 \frac{30 \times 10^6}{18.6 \times 60}$$

$$= 247 \text{ pound feet}^2$$

$$= 267$$

Dec 14 1930
 Hoop, Elmer, Jr.,
 Chas. Kingsley, Jr.,
 Movies of Paul
 J. Outt.

Synchronizing tests
 Movies of pulling
 into step phenomena.

No.	P_{KW}	I_f	I_m	Remarks.		
1	0	0		regular start,		
2	33 KW	185V	9.6	15.5	pullin ok. after start.	10ft
3	36	195	10.0	15.5		10ft
4		ditto		15.0		5ft
5		ditto		15		5ft
6		ditto		14 amp.	failed to pull in.	8ft
7				14.	pullin ok.	
8	30.5	185	10.0	13.9	Did not syn.	
9.	35.5	185	10.0	13.3	" " "	
10.	28.0	170	8.5	12.2	two shots	
11.	0	0		10±2	Brought up to step pulling as reflect motor with wrong polarity.	

Two 1000s with lamps were hung above the set and were used in taking movies of the starting. Mr. Kingsley posed for the picture at the electric panel.

The film speed was 16 frames per sec. and opening F 1.9. 16mm film Panchromatic. Eastman.

Dec 15 1930. These films came back last Thursday day and they were fine. I put on a show Friday evening in the lab.

Notebook # 3

Filming and Separation Record

_____ unmounted photograph(s)

1 negative strip(s) *inside mounted envelope pg. 51*

_____ unmounted page(s)
(notes, drawings, letters, etc.)

was/were filmed where originally located between page 50 and 51.

Item(s) now housed in accompanying folder.



Moving Picture Photographs of 10 pole
Synchronous motor. 1.9 lens 16 f.p.s.
Panchromatic film. by Henry Lane.

For all strobograms of Jan 7¹⁹³¹ by Prof. Bowles.

33 KW 10 amps \pm field current
shaft load

Kane Wilder.

3-4616

Dec 4 1982.

March 19, 1931

W.E. Hunt

Integrgraph Solutions of Salient Pole Motor.

$$\frac{P_e}{P_m} = k \frac{d\theta}{dt}$$

$$= \frac{1.20}{0.2} = 7.5$$

K. 0.177

Reluctance = $0.3 \times P_m$. $b = 0$

Chart #1

	$\frac{P_e}{P_m}$	k	θ_0	constant $\frac{d\theta}{dt}$	
	0.15	.02		$\frac{1.20}{7.5}$	Initial Steady State.
	.15	.0167	0		● 55° - 160
	.20	.02	0	$\frac{.95}{10.0}$	
.176	(.167)	.0167		.95	42° ● 75 - 150±
				.95	
.263	.25	.30	.02	0	55/15. 0 unstable very!
.22	.208	.25	.02	0	.74/12.5 ● 103 - 149 stable.
.238	.225	.27	.02	0	.60/13.5 ● 122 -
.245	.233	.28	.02	0	.63/14 ● very close to critical just stable.
.250	.237	.285	.02	0	○ unstable.

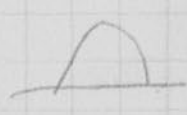
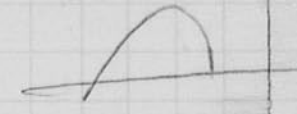

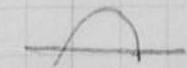
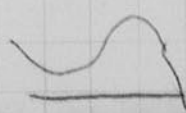
k = .0167 instead of .02
 Wrong

March 20 1930

Ditto,

K=0177

Chart 2

P4Pm	k	θ_0	$\frac{d\theta}{dt}$ var.	def. along	
.132	1.5 02 0167	180	1.2 1.34	7.5	○
.123	1.4	180	1.2	7.0	○
.105	1.2	180		6.0	●
.114	1.3	180	1.3	6.5	○
2.6					backwards 330° 32
2.3.			.72 .82		332 29. backwards
2.6	02				"
2.0					" 
1.7					" 
1.4					" 
1.6.					" 
1.5					" 

From Chart 2.

 $\frac{P_e}{P_m}$ Q_0

.14

- 238
100

.17

264
74

.20

290
63

Chart 3,

Mar 21 1931

Edgents

Brown & Green

$\frac{P_L}{P_m}$	θ_0	k	
.56	0	.045	●
.565	0	"	○
.38	180	"	○
.375	180	"	○
.370	180	"	○
.360	180	"	●

 $K = 0.475$

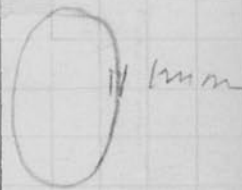
critical at .365?

A.A.

Chart 3a

rest of Integrator.

$$k = 0$$

 $\frac{P_L}{P_m} = 1$ slope through 90° straight line for $f(\theta)$ force

.34	0	.03	●
.40	0	.03	○ ●
.39	0	.03	
.42	0	.03	○

 $K = 0.318$ Rounded Rotor. $P_R = 0$.

4.5 about critical.

March 28 1931.

Chart 4.

R = 0635

New Integrator setup II

$\frac{P_L}{P_m}$	θ_0	$\frac{d\theta}{dt_{max}}$	h	$\frac{d\theta}{dt}$	inch $\frac{d\theta}{dt}$	
.68	0		0.06	11.33	6.01	●
.70	0		"		6.18	○
.69	0		"		6.09	●
.53	180				4.68	

Mar 29 1931.

Chart 5

Integragraph setup V.

on slip table
.375 in = 1 unit of slip.

P_2/P_1	θ_0	k	$\frac{d\theta}{dt}$ units	$\frac{d\theta}{dt}$ inches	
.715	0	.06	11.59	4.46	0.
.690	0	.06	1	4.32	0
.53	180	.06		3.315	0
.52	"	.06		3.25	0
.51	"	.06		3.19	very close 0

Reverse Solutions -

Chart 6

$R = .06$

P_2/P_1	θ	k	$\frac{d\theta}{dt}$	$\frac{d\theta}{dt}$	θ
.67		.63	.59	.55	.52

116 (final)	118	121.5	124	127 (final angle)
36	52	67	72	77
340	310	281	254	227
				112

P_2/P_1	θ	k	$\frac{d\theta}{dt}$	$\frac{d\theta}{dt}$ inches
-----------	----------	---	----------------------	--------------------------------

.53	77°	.06	3.315	• barely
.53	80°	.06	"	•• second swing in just
.54	77.5	.06	3.38	••• out on 3rd swing - just

Round Rotor Solution.

.70	0	.06	4.37
-----	---	-----	------

Chart 7. Setup V.

P_c/P_m	k	θ	$\frac{d\theta}{dt}$ mm.	$\frac{d\theta}{dt}$ in.	
.84	.085	0		3.95	● ?
.82	"	0		3.855	●
.85	"	0		3.995	○ close.
.72	"	180		3.38	○
.68		180		3.70	○ close
.67				3.15	●

The scale on the θ table
was in error for the solutions
the k slope is correct on this page
but needs to be corrected by

$$\frac{1}{\sqrt{\frac{8.45}{9.39}}} = \frac{\cancel{1.058}}{1.058} = 1.058$$

for all values up to now

that is k as noted
should be $1.058 k$.

April 1, 1931.

Determination of the conditions
 wherein a Salient-Pole Synchronous
 Motor pulls into step as a Reluctance
 Motor.

is The equation stating the restrictions

$$P_j \frac{d^2 \theta}{dt^2} + P_d \frac{d\theta}{dt} + P_r \sin 2\theta = \underline{P}_L$$

First change variables so that $2\theta = \alpha$

$$P_j \frac{d^2 \left(\frac{\alpha}{2}\right)}{dt^2} + P_d \frac{d\left(\frac{\alpha}{2}\right)}{dt} + \underline{P}_r \sin \alpha = \underline{P}_L$$

$$P_j \frac{d^2 \alpha}{dt^2} + P_d \frac{d\alpha}{dt} + 2\underline{P}_r \sin \alpha = 2\underline{P}_L$$

Change time variable so that $t = \frac{\tau}{a}$.

$$a^2 P_j \frac{d^2 \alpha}{d\tau^2} + a P_d \frac{d\alpha}{d\tau} + 2\underline{P}_r \sin \alpha = 2\underline{P}_L$$

$$\frac{d^2 \alpha}{d\tau^2} + \frac{a P_d}{a^2 P_j} \frac{d\alpha}{d\tau} + \frac{2\underline{P}_r \sin \alpha}{a^2 P_j} = \frac{2\underline{P}_L}{a^2 P_j}$$

now let $\frac{2\underline{P}_r}{a^2 P_j} = 1$

$$a = \sqrt{\frac{2\underline{P}_r}{P_j}}$$

$$\frac{d^2 \alpha}{d\tau^2} + \frac{P_d}{P_j \sqrt{\frac{2\underline{P}_r}{P_j}}} \frac{d\alpha}{d\tau} + \frac{2\underline{P}_r \sin \alpha}{\frac{P_j \cdot 2\underline{P}_r}{P_j}} = \frac{2\underline{P}_L}{\frac{P_j \cdot (2\underline{P}_r)}{P_j}}$$

$$\frac{d^2 \alpha}{d\tau^2} + \frac{P_d}{\sqrt{2\underline{P}_j \underline{P}_r}} \frac{d\alpha}{d\tau} + \sin \alpha = \frac{\underline{P}_L}{\underline{P}_r}$$

From the cylindrical rotor
case then with

$$k \text{ as } \frac{P_d}{\sqrt{2P_j P_R}} \text{ and } \frac{P_e}{P_R}$$

it can be found whether or
not the conditions fall above
or below the ultimate line
for synchronization.

~~With $P_R = 2.3$~~

Unbalanced-Rotor Induction Motor

The torque equation for an induction motor with an unbalanced rotor is of this form

$$P_j \frac{d^2\theta}{dt^2} + D(1 - b \cos 2\theta) \frac{d\theta}{dt} = P_L$$

$$\frac{d^2\theta}{dt^2} + \frac{P_d}{P_j}(1 - b \cos 2\theta) \frac{d\theta}{dt} = \frac{P_L}{P_j}$$

or since $\frac{d\theta}{dt} = s$,

$$\frac{ds}{dt} + \frac{P_d}{P_j}(1 - b \cos 2\theta) s = \frac{P_L}{P_j}$$

or in another form by changing the units of time
let $t = \tau/a$ $a = \text{constant}$.

$$a^2 P_j \frac{d^2\theta}{d\tau^2} + a P_d (1 - b \cos 2\theta) \frac{d\theta}{d\tau} = P_L$$

$$\text{Let } a^2 P_j = 1$$

$$a = \frac{1}{\sqrt{P_j}}$$

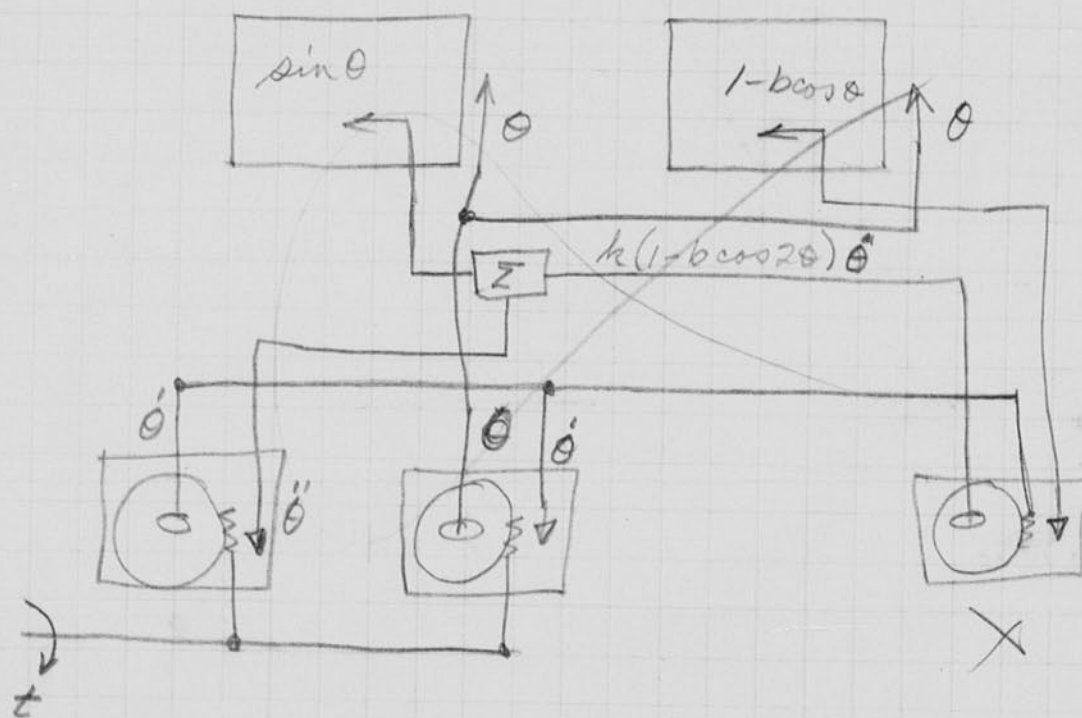
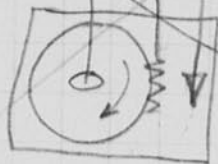
$$\frac{d^2\theta}{d\tau^2} + \frac{P_d}{\sqrt{P_j}}(1 - b \cos 2\theta) \frac{d\theta}{d\tau} = P_L$$

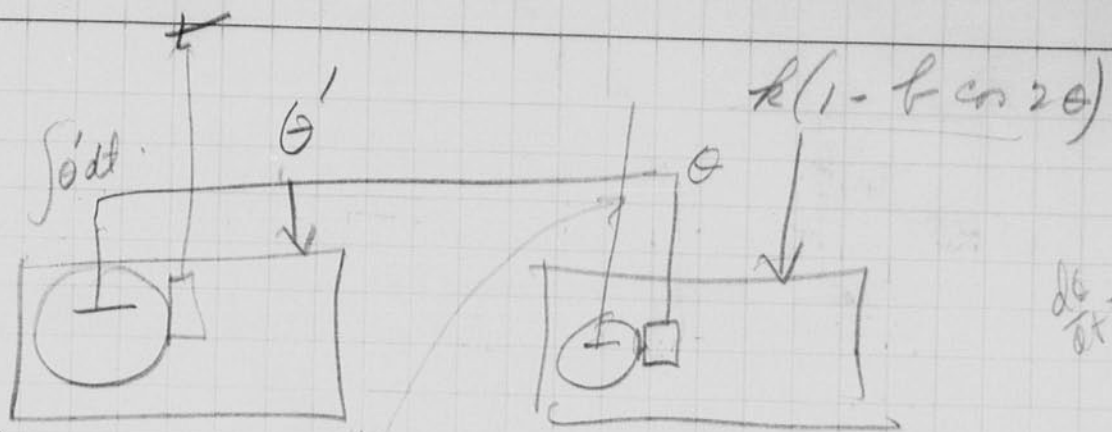
Steady state

$$\ddot{\theta} + k(1 - b \cos 2\theta) \dot{\theta} = P_L$$

$$\theta = \int_0^t \int_0^t [P_L - k(1 - b \cos 2\theta) \dot{\theta}] dt dt$$

$$\dot{\theta}(1 - b \cos 2\theta) \quad \dot{\theta}(1 - b \cos 2\theta)$$





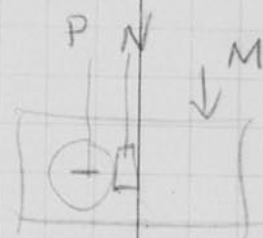
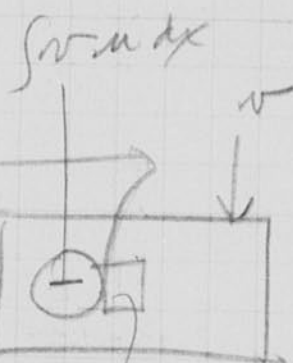
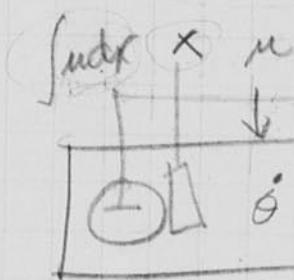
$$\frac{d\theta}{dt} = \dot{\theta}$$

$$\theta = \int \dot{\theta} dt$$

$$\dot{\theta} = \int d\theta$$

$$\int k(1 - b \cos 2\theta) \dot{\theta} dt$$

$$\int u v dx$$

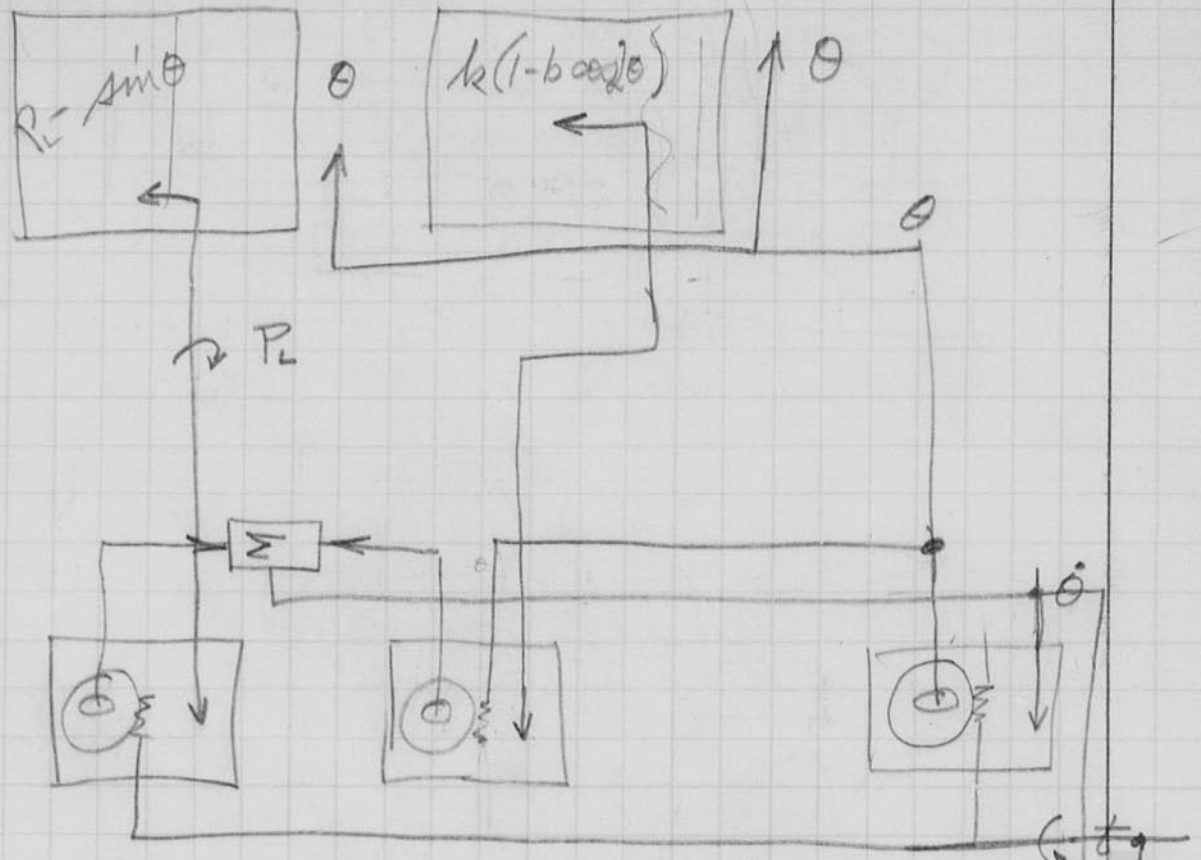


$$P = \int M dN$$

$$\theta = \iint P_L$$

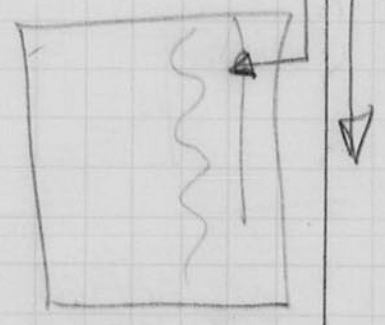
$$\int u dx \quad du \int u dx = u \cdot x$$

$$\theta = \int \left[\int P_L dt - \int k(1 - b \cos 2\theta) \dot{\theta} dt \right] dt$$



$\int P_L dt$ $\int k(1 - b \cos 2\theta) dt$

$\int k(1 - b \cos 2\theta) d\theta$



Notebook # 3

Filming and Separation Record

1 unmounted photograph(s)

 negative strip(s)

 unmounted page(s)
(notes, drawings, letters, etc.)

was/were filmed where originally located between page 64 and 65.

Item(s) now housed in accompanying folder.

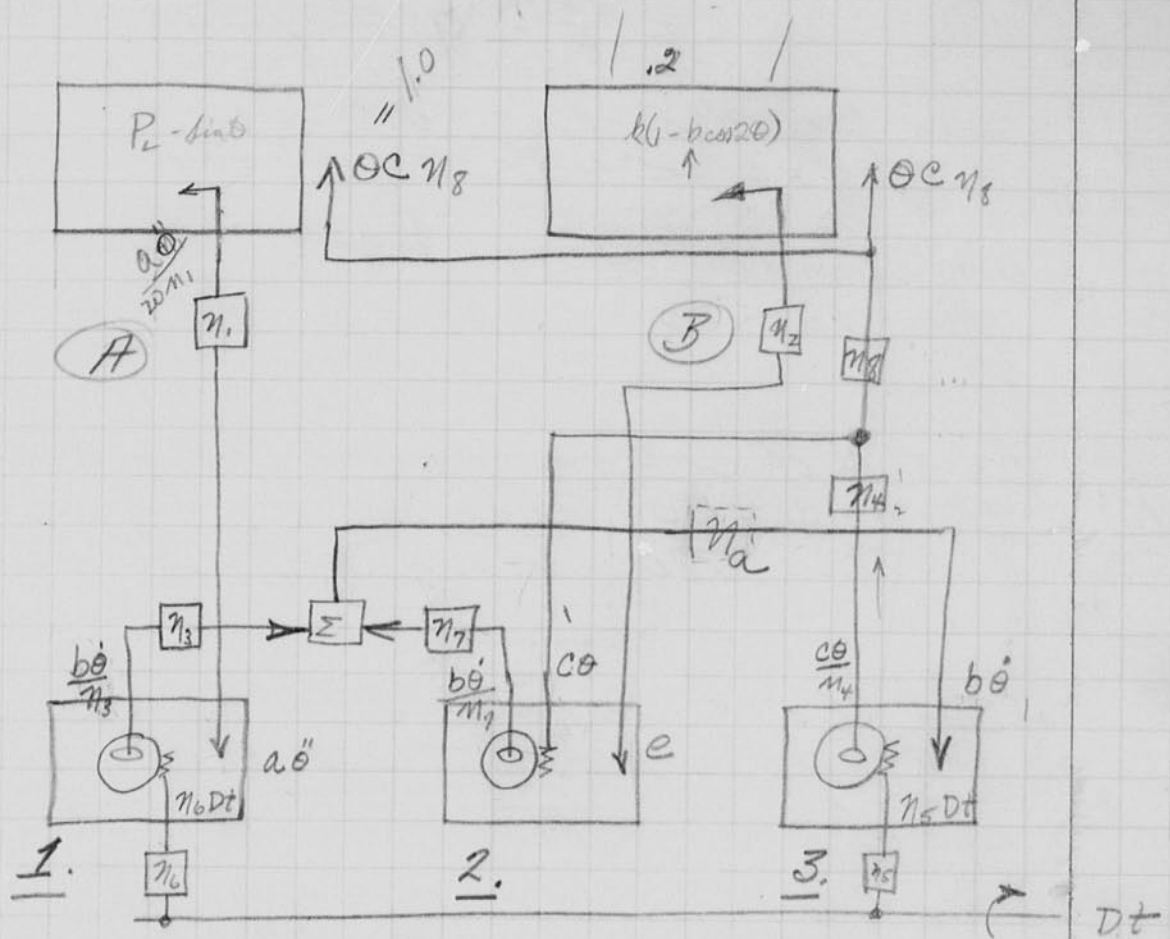
Apr



Dt

Determination of Scales for
non-linear damping dyn. mach. Prob.

April 5/1931.



Integrator limitations

$$a\ddot{\theta} < 40 \text{ rev.}.$$

$$e < 40 \text{ "}$$

$$b\dot{\theta} < 40 \text{ rev.}$$

$$c\theta < 400 \text{ rev. per min.}$$

Unit relationships

$$1 \quad \frac{b\dot{\theta}}{\eta_3} = \frac{1}{32} \int a\ddot{\theta} \eta_6 D dt \rightarrow 1 = \frac{\eta_3 \eta_6 a D}{32 b}$$

$$2. \quad \frac{b\dot{\theta}}{\eta_7} = \frac{1}{32} \int e c d\theta \rightarrow 1 = \frac{\eta_7 e c}{b 32}$$

$$3. \quad \frac{c\theta}{\eta_4} = \frac{1}{32} \int b\dot{\theta} \eta_5 D dt. \quad 1 = \frac{\eta_4 \eta_5 b D}{32 c}$$

$\dot{\theta}_{\max}$ is about 2 or less
 so $a < \frac{40}{2} = 20$ rev/unit Let $a = 16$

max for e units = 0.2
 $e \times 0.2 < 40$ $e < \frac{40}{0.2} = 200$ Let $e = 128$

$\dot{\theta}_{\max}$ is about 20
 $b\dot{\theta} < 40$ $b < \frac{40}{20} = 2$ let $b = 2$.

Plot of chart on No A table so
 that $c = 1.0$ also let $n_8 = 1.0$
 then 180 elect deg. = 9.0 inches

Trial II Unit equations become

$$1. \quad 1 = \frac{n_3 n_6 a D}{32 b} = \frac{n_3 n_6 D}{4}$$

$$n_7 = 1/2$$

$$2. \quad 1 = \frac{n_7 e c}{b 32} = 2 n_7$$

$$3. \quad 1 = \frac{n_4 n_5 b D}{32 c} = \frac{n_4 n_5 D}{16}$$

There are left 5 unknowns and two equations
 so that three quantities may be
 selected at random.

$$n_3 = 1/2$$

$$n_4 = 1/2$$

$$n_5 = 1$$

$$D = 32$$

$$n_6 = 1/4$$

$$\text{Let } n_3 = 1/2 \quad n_4 = 1/2 \quad n_5 = 1.$$

$$\text{from 3.} \quad 1 = \frac{1/2 \cdot 1}{16} D \quad D = 32$$

$$\text{from 1} \quad 1 = \frac{1/2 \cdot n_6 \cdot 32}{4} \quad n_6 = 1/4$$

Check.

$$1. \quad 1 = \frac{n_3 n_6 a D}{32 b} = \frac{1/2 \cdot 1/4 \cdot 16 \cdot 32}{32 \cdot 2} = 1 \checkmark$$

$$2. \quad 1 = \frac{n_7 e c}{b 32} = \frac{1/2 \cdot 128 \cdot 1}{2 \cdot 32} = 1 \checkmark$$

$$3. \quad 1 = \frac{n_4 n_5 b D}{32 c} = \frac{1/2 \cdot 1 \cdot 2 \cdot 32}{32 \cdot 1} = 1 \checkmark$$

Units (vertical) on B table.

$$e = 128 \text{ revs/unit.}$$

$$\text{Let } \eta_2 = 1/8$$

$$\frac{e}{20\eta_2} = \frac{128}{20 \times \frac{1}{8}} = \underline{51.2 \text{ inches per unit.}}$$

$$\eta_2 = 1/8$$

Units (vertical) on A table.

$$a = 16 \text{ revs/unit.}$$

$$\text{Let } \eta_1 = 1/8$$

$$\frac{a}{20\eta_1} = \frac{16}{20 \times \frac{1}{8}} = \underline{6.4 \text{ inches/unit.}}$$

$$\eta_1 = 1/8$$

Horizontal scale on A and B tables is the same and is equal to

$$\underline{\underline{9'' = 180 \text{ elect degrees}}}$$

trial VII In case $k = 0.05$ or less, the e scale can be doubled to 256 rev/unit.

The vertical scale would then become

$$\frac{e}{20\eta_2} = \frac{256}{20 \times \frac{1}{8}} = 102.4 \text{ inches/unit.}$$

Since e only appears in unit equat 2 and since c and b are fixed, η_2 needs to be changed in the opposite sense to e

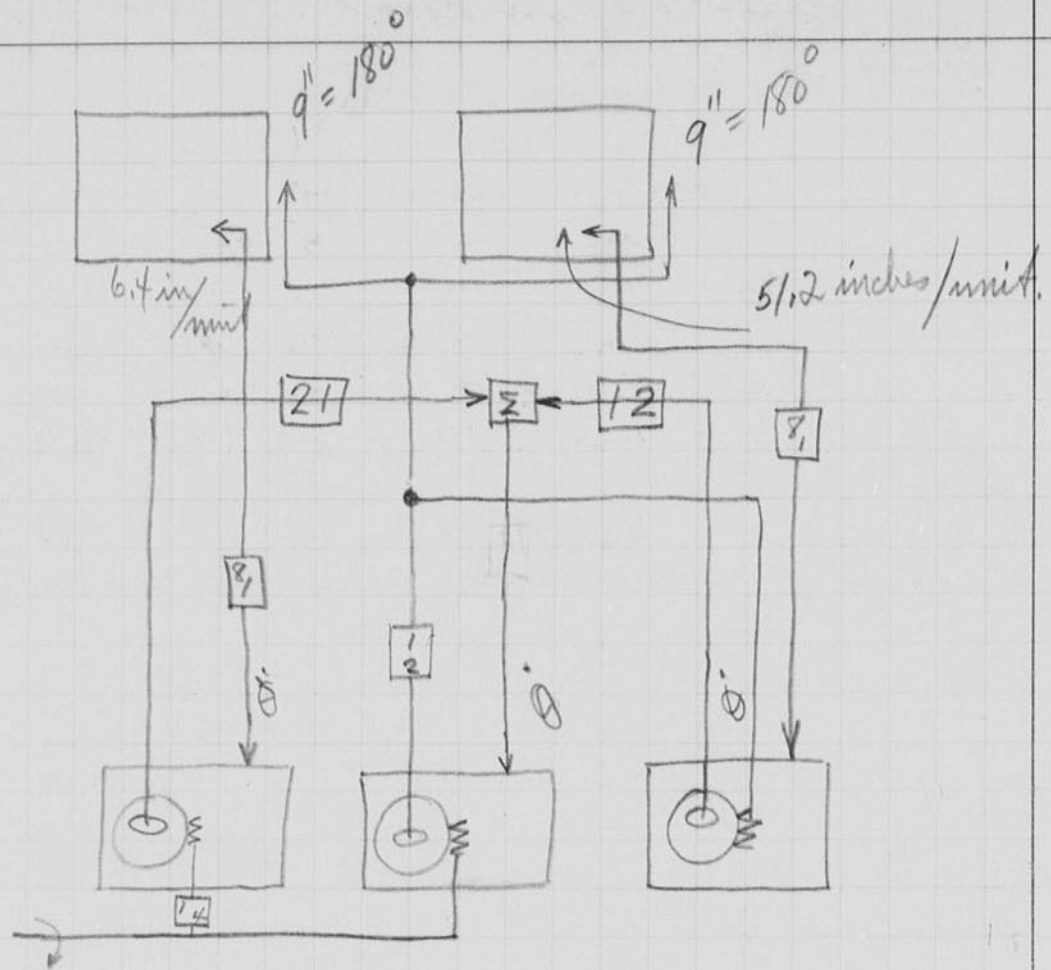
$$\eta_2 = 1/4$$

$$\textcircled{2} \quad 1 = \frac{\eta_2 e c}{532} = \eta_2 \frac{256 \cdot 1.0}{32 \times 2} = \eta_2 4 \quad \eta_2 = 1/4$$

$$\text{or } e = \frac{512}{20 \cdot 8} \text{ for } \eta_2 = 1/8 \text{ scales VII}$$

$$\begin{array}{l} \times e = 204.8 \quad \text{for } \eta_2 = 1/4 \\ \times e = 102.4 \quad \text{for } \eta_2 = 1/2 \end{array}$$

scales
I



Setup for scales VI

Effect of the time constant
of the field circuit.

April 5, 1931.

The inductance of the field circuit will retard the buildup of the synchronizing torque in approximately an exponential relationship. That is the synchronizing force is actually more closely represented by $[P_m (1 - e^{-\alpha t}) \sin \theta] 1$ than $(P_m \sin \theta) 1$.

$$P_j \frac{d^2 \theta}{dt^2} + \frac{P_d}{P_j} \frac{d\theta}{dt} + [P_m (1 - e^{-\alpha t}) \sin \theta] 1 = P_L$$

$$\frac{d^2 \theta}{dt^2} + \frac{P_d}{P_j} \frac{d\theta}{dt} + \left[\frac{P_m}{P_j} (1 - e^{-\alpha t}) \sin \theta \right] 1 = \frac{P_L}{P_j}$$

Let $t = \frac{\lambda}{a}$ where $\lambda =$ a new variable.

$$a^2 \frac{d^2 \theta}{d\lambda^2} + \frac{a P_d}{P_j} \frac{d\theta}{d\lambda} + \left[\frac{P_m}{P_j} (1 - e^{-\frac{\alpha \lambda}{a}}) \sin \theta \right] 1 = \frac{P_L}{P_j}$$

$$\frac{d^2 \theta}{d\lambda^2} + \frac{P_d}{a P_j} \frac{d\theta}{d\lambda} + \left[\frac{P_m}{a^2 P_j} (1 - e^{-\frac{\alpha \lambda}{a}}) \sin \theta \right] 1 = \frac{P_L}{P_j a^2}$$

Now Let $\frac{P_m}{a^2 P_j} = 1$

or $a = \sqrt{\frac{P_m}{P_j}}$

which gives

$$\frac{d^2 \theta}{d\lambda^2} + \frac{P_d}{\sqrt{P_j P_m}} \frac{d\theta}{d\lambda} + [(1 - e^{-\frac{\alpha \lambda}{a}}) \sin \theta] 1 = \frac{P_L}{P_m}$$

$$\theta = \iint \left\{ \frac{P_L}{P_m} - k \frac{d\theta}{d\lambda} - [(1 - e^{-\frac{\alpha \lambda}{a}}) \sin \theta] 1 \right\} d\lambda d\lambda$$

$$= \int \left\{ \int \frac{P_L}{P_m} d\lambda - \int k \frac{d\theta}{d\lambda} d\lambda - \int (1 - e^{-\frac{\alpha \lambda}{a}}) \sin \theta d\lambda \right\} d\lambda$$

Slip variations due to a Reluctance torque
equal to .3 P_m. Values taken from charts 1 to P_{inc}.

Chart	L.R.	k.	θ_{max}	θ_{min}	$\frac{\theta_{max} - \theta_{min}}{2}$	$\frac{\theta_{max} + \theta_{min}}{2}$	$\frac{\theta_{max} - \theta_{min}}{\theta_{max} + \theta_{min}}$ % var.	$\frac{\theta_{max} - \theta_{min}}{\theta_{max} + \theta_{min}}$ % var.
1. p. 52.	.15	0.177	1.62	1.4	.22	1.4	15.7	16.
	.20	"	2.05	1.88	.17	1.88	9.05	9.5
	.25	"	2.48	2.34	.14	2.34	5.98	5.92
	.27	"	2.68	2.54	.14	2.54	5.52	4.45
	.28	"	2.76	2.63	.13	2.63	4.95	4.5
	.30	"	2.93	2.80	.13	2.80	4.65	3.67
	.15	"	1.63		.22	1.41	15.6	16.0
	.14	"	1.55		.24	1.31	18.35	19.4
	.13	"	1.46		.25	1.21	20.65	20.0
	.12	"	1.38		.28	1.10	25.40	—
2.	.14	"	3.48	2.61	.40	3.08		13.0
	.17	"	4.09	3.38	.34	3.75		9.07
	.20	"	4.70	4.09	.30	4.40		6.82
	.23	"	5.30	4.77	.24	5.06		4.75

April 8 1931

Solutions with set up of page 68. scales VI

Cylindrical rotor check.

51.2
204.8

P_r/P_m	h_c	θ_0	θ_0'	K		
0.6	.05	12.0	0	●	close.	Perfect check of Brown & Jameshausen
0.605	.05	12.1	0	○	close.	
		6	2X0	512K	0.	
.89	.12	7.42	14.84	6.14	180	● close
.90	.12	7.50	15.0	"	180	○ closer.
.915	.12	7.62	15.24	"	180	critical.
.97	.14	6.93	13.86	7.2	180	θ_{max} 24.75 ● close.
.98	.14	7.0	14.0	7.2	180	● closer.

η changed to $1/2$ from $1/2$ scale thereby modified. Scales VI
(see page 67). 204.8 inches per inch

Chart 8

.14	.01	14.0	28.0	2.064	0	●	30°
.15	.01	15.0	30.0	"	0	○	
.145	.01	14.5	29.0	2.064	0	○	close.
.06	.01	6.0	12.0	2.064	180	θ_{max} 31.3	○
.05	.01	5.0	10.0	2.064	180		● 10°
.055	.01	5.5	11.0	2.064	180	31.75	● 1° close

Spherical Pole Case $P_r = 0.3 P_m$

.055	.01	5.5	11.0	2.064	180	32.3	○ Initial slip = 14.25
.05	.01	5.0	10.0	2.064	180		○ second ●
.04	.01	4.0	8.0	"	180	31.3	● " 158°
.14	.01	14	28	"	0		○
.13	.01	13	26	"	0		● 45°
.135	.01	13.5	27	"	0		●

Chart 9

.38	.03	12.66	25.32	6.195	0		○
.37	.03	12.32	24.64	6.190	0		● 15°
.21	.03	7.0	14.0	"	180	31	● 13°
.22	.03	7.3	14.6	"	180		○
.38	.05	7.60	15.2	-10.32	180		●
.39	.05	7.8	15.4	"	"		○? Shifting final pt.

Changes of scales. Diagram on page 65

Given constants.

$$a = 16$$

$$e = \frac{128}{32} \text{ (Depends upon scales).}$$

$$b = 2$$

$$c = 2 \quad m_8 = 1/2 \text{ (these were } c = 1 \quad m_8 = 1).$$

The angle scales and plots remain the same as scales II and III.

$$\text{Let } m_a = 1/4$$

This gives a 4:1 scale relationship between the output slip on the resultant table of this VIII against that of II and III.

Unit equations

$$1 = \frac{m_3 m_a m_b a D}{32b} = \frac{m_3 m_b 16 D}{4 \cdot 32 \times 2} = \frac{m_3 m_b D}{16}$$

$$1 = \frac{m_7 m_c e c}{6 \times 32} = \frac{m_7 e 2}{4 \times 2 \times 32} = \frac{m_7 e}{128}$$

(Let $c = \frac{512}{204.8}$ as in III) or 204.8 in/unit.

$$\text{then } m_7 = \frac{128}{512.0} = \frac{1}{4}$$

$$1 = \frac{m_4 m_5 b D}{32c} = \frac{m_4 m_5 2 D}{32 \times 2} = \frac{m_4 m_5 D}{32} =$$

$$\text{Let } m_5 = 1 \quad m_4 = 1/2 \quad m_3 = 1/2$$

$$m_4 m_5 D = 32 \quad D = 32 \times 2 = 64$$

$$m_6 = \frac{16}{m_3 D} = \frac{16}{64 \cdot 1/2} = \left(\frac{1}{2} \right)$$

April 11, 1931.

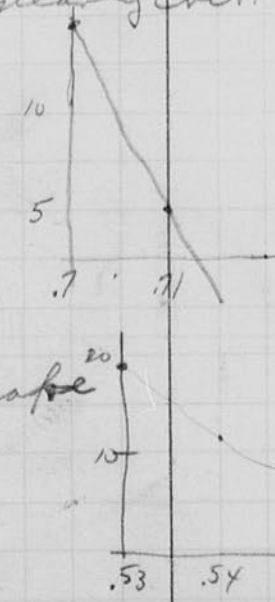
(b=1) Slip variations.

	k	$\frac{r}{p_{max}}$	$\frac{d\theta}{dt}$	$\frac{d\theta}{dt}$ sum of 5 trials	variation in θ $\theta_{min} - \theta_{avg}$	θ_{min}	θ_{avg}	θ_{max}		
9	.03	.1	3.33	6.67 $\rightarrow 241^\circ$	$1. \frac{370}{360}$	$1. \frac{170}{360}$	1.935	1.389	2.30	2.03
	.03	.2	6.66	13.32 120	$1.207/360$	$1. \frac{253}{360}$	1.830	1.647	13.60	12.36
	.03	.3	10.	20.0	$1. \frac{287}{360}$	$1. \frac{220}{360}$	1.778	1.611	23.5	20.5
	.03	.4	13.33	26.66	$1. \frac{287}{360}$	$1. \frac{222}{360}$	1.777	1.617	26.4	26.6
	.03	.5	16.67	33.34	$1. \frac{245}{360}$	$1. \frac{240}{360}$	1.689	1.666	5.65	5.0
10	.05	.15	10.0	20.0	$\frac{310}{360}$	0.861	$\frac{300}{360}$	0.833	4.30	4.16
	.05	.095	5	10.0	$\frac{274}{360}$.761	$\frac{284}{360}$.789	1.17	1.20

Slip variation due to Rub distance torque 3 P.m.

	k	$\frac{r}{p_{max}}$	$\frac{d\theta}{dt}$	$\frac{d\theta}{dt}$	θ	Notes
11	.03	.15	5.0	10	θ_0	• Very stable.
11	.03	.23	7.67	15.34	θ_0	•
	.03	.39	13	26.	0	•
	.03	.42	14	28.	0	•
	.03	.40	13.33	26.66	0	•
	.03	.41	13.66	27.34	0	•
12	.03	.23	7.67	15.333	180	• very.
	.03	.24	8.0	16.0	180	• close. 40
	.03	.25	8.33	16.66	180	• close 7° nearly crit.
	.03	.26	8.66	17.33	180	•
13	.06	.70	11.67	23.33	0°	• 14°
	.06	.71	11.83	23.65	0°	• 5°
	.06	.72	12.0	24.0	0°	• 2° close
	.06	.73	12.18	24.33	0°	•
	.06	.53	8.83	17.66	180°	• 18° to space
	.06	.54	9.0	18.0	180°	• 12
	.06	.55	9.17	18.33	180°	• 8
.06	.56	9.33	18.66	180°	•	

change $e = \frac{517}{2}$ $11.7 = 1/2$





April 12, 1931.
D. E. Edgerton.

Reverse solutions.

$$R = 0.06 \quad P_R = 0.3 P_m.$$

Chart.	k	L.R.	θ FINAL.	θ start		$\dot{\theta}$ ang. turn	
14	0.06	.48	129.5	128.	✓	8.	16
"		.51	128.0	126.5	✓		17
"		.54	125.5	124.0	✓		18.
"		.57	123.5	122.0	✓		19
"		.50	121	119.5	✓		20.
"		.63	119	117.5	✓		21.
"		.66	117	115.5	✓		22
"		.69	114.5	113.0	✓		23.
Forward solution							
"	.63			$0^\circ \theta_0$			o very stable? !!

Forward check for cylindrical rotor
0.06 .69 nearly critical o just barely Good check!

Some trouble was experienced with the integrator due to too much load on the angle shaft. This may explain some of the discrepancy of the days results.

April 14, 1931. Data from Integraph Solutions.

Chart no.	k.	θ_0	L.R.	$\dot{\theta}_0$ avg. inches	$\dot{\theta}_0$ inst. inches	$\dot{\theta}$ max. transient	$\theta=180^\circ$	$\theta=0^\circ$
							$\frac{\dot{\theta}_{max}}{\text{Avg.}}$	$\frac{\dot{\theta}_{max}}{\text{Avg.}}$
1	.0177	180	.12	1.13	1.38	3.1	2.75	1.22
2	.0475	180	.36	3.0	3.38	6.05	2.02	1.13
5	.0635	180	.51	3.22	3.59	6.0	1.87	1.12
7	.085	180	.61	3.16	3.52	5.7	1.80	1.11
8	.01	180	.04+	.41	.66	1.56	3.32	1.40
9	.03	180	.21	.71	.805	1.56	2.20	1.14
9.	.05	180	.38	.76	.88	1.52	2.0	1.16
20	.10	180	.795	.32	.36	5.61	1.77	
19	.02	180	.13	.247	—	6.3	2.55	

Chart 14. intersections. $k = 0.06$
 $P_R = 0.3 P_m.$

L.R.	Steady State				transient $\dot{\theta}_{max}$	θ_0	θ_0
	$\dot{\theta}$ average	$\dot{\theta}_{max}$	$\dot{\theta}_{min}$	$\frac{\dot{\theta}_{max}}{\dot{\theta}_{min.}}$ SS			
.48	3.2		3.6			93	
.51	3.4		3.8			76°	240
.54	3.6		3.95			70	262
.57	3.8		4.18			60.5	281
.60	4.0		4.35			31.5	298.5
.63	4.2		4.51			31.5	314
.66	4.4		4.72			16.5	342
.69	4.6						

April 18-1931
H.S. Edgerton

Integragraph Solutions.

Setup as outlined on page ~~72~~ 73

Damping scale = $\frac{204.8}{102.4}$ inches/unit on table ~~2~~ 2.
Reverse solutions. $M_7 = 1/2$.

Chart	h.	L.R.	FINAL	Start	θ avg.	θ turns
14.15	0					
	0.06	.49	129	127.5	8.17	16 $\frac{1}{3}$
	.06	.50	127.5	126.0		16 $\frac{2}{3}$
	.06	.51	127 128	125.5 126.5		16 $\frac{2}{3}$
	.06	.50	127.5	126.0		

0.06
1.024
0.06
0.144
16 x .49 = .784

Speed variations due to reluctance torque.

16	0.06	.3	5	10
		.6	10	20
		.9	15	30.

April 19, 1931. Cont of speed variations

0.04	.3	2	5	7.5	15	10
	.6	.4	10		20	
	.9	.6	15		30	

Pulling into step equa

	h.	L.R.	θ	θ turns	Notes
17.	0.06	.48	180°	8 16.	o just! .475 probably stable.
	.06	.65	0	10.84 2.65 2.30	• quite stable
	.06	.67	0	22.30 11.0	o ? belt off.

Reverse Solutions

$P_r = 0.3 P_m$

$b = 0$

Chart

	h	LR	θ_{final}	θ_{start}	θ_{avg}	$\dot{\theta}_{turns}$	θ_0	θ_1
17	0.04	.32	141.5	140	8	16	98°	225
		.36	139.	138.5	9	18	49.0	256
		.40	135	133.5	10	20	69.0	283
		.44	132	130.5	11	22	48.5	307
		.48	128.5	127.5	12	24	13°	345
		.34	140.5	139.0	8.5	17.	86.0	243

Forward tests

8.18 inch

18

	h	LR	θ_0	θ_{turn}	$\dot{\theta}$	θ
0.08	.64	180°	8	16	● very close 2 or 3°	
08	.645	180	8.07	16.14		
				47°		
08	.81	0	10.13	20.26	●	50
	.82	0		20.50	0	
	.82	0				

Apr. 20, 1931.

Reverse Solutions

$$6.14 \frac{0.06}{0.02} = 2.045$$

Chart	k.	L.R.	θ final	θ start	$\dot{\theta}$ average	$\dot{\theta}$ turns.	θ_0	θ_1
	0.02.	275						
19.	.26 ✓	147.5	146	13	26 ✓	27.0	339	
	.24 ✓	149.5	148	12	24 ✓	48.5	310	
	.22 ✓	151.5	150	11	22 ✓	65	290	
	.20 ✓	153.5	151.5	10	20 ✓	77.5	274	
	.18 ✓	155.5	154.0	9	18 ✓	89	258	
	.17 ✓	"	155.0		17 ✓	94	249	
	.16 ✓	158	156.5	8	16 ✓	99	240	
	.15	159	157.5	7.5	15			
	.14 ✓	160	158.5	7	14 ✓	108	220	
	.13	161.5	160.0	6.5	13.			

Forward solutions.

$$6.14 \frac{1}{.6} = 10.23$$

	k.	L.R.	$\dot{\theta}$ turns	θ_0		
20.	0.10	0.91	9.1	18.2	0	● 12°
		.92	9.2	18.4	0	● 8°
	.10	.94	9.4	18.8	0	○ critical 93.5
						○ just
Apr 21.	.10	.8	8.0	16.0	180	○ just.
	.10	.79	7.9	15.8	180	● just! 1° to go.

$$\frac{d\theta}{dt} = 16 \text{ units}$$

$$D = 64 \text{ rev/unit.}$$

$$\frac{64}{400} =$$

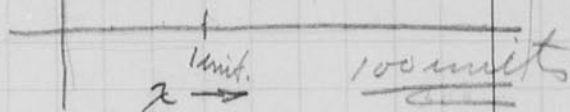
$$\begin{aligned} \frac{64 \text{ rev/unit}}{16} &= 4 \text{ rev/unit.} \\ &= \frac{4}{20} \text{ inch/unit. ?} \\ &= \frac{1}{5} \text{ in/unit} \\ &\text{or } 5 \text{ units/inch} \end{aligned}$$

Results against time.

$$6.14 \times \frac{4}{6} = 4.095$$

h	L.R.	θ_0	$\dot{\theta}_{avg}$	turns		
.04	.31	180	7.75	15.5	○	
.04	.30	180	7.5	15.0	○	
	.29	180	7.25	14.5	●	14°
.04	.48	0	12	24	●	8°

$$\frac{d\theta}{dt} = 16$$



$$100 \times 64 = 6400 \text{ rev.}$$

$$\Rightarrow \frac{6400}{20} = \text{inch} = 20''$$

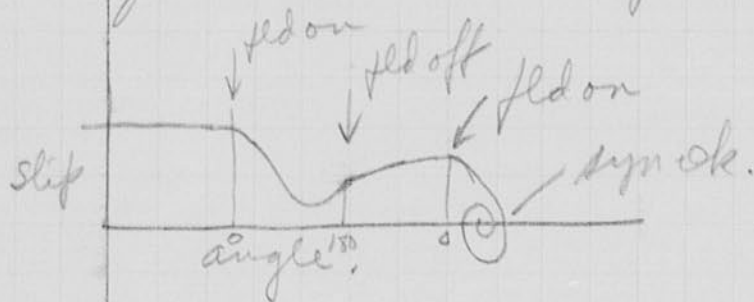
$$\eta = \frac{400}{6400} = \frac{1}{16}$$



April 1, 1931.
W. J. ...

Pulling into step with
intermittent field current.

If the exciter is connected only when the field is such as to give motor action it might be possible to pull into step heavier loads. The stroboscopic relay can be arranged so that the field circuit is energized only when the angle lies between 0 and 180 electrical degrees. Then if a motor fails to pull in on the first swing, it may will not be driven further from synchronism when the angle is such as to give generator action since the field will be opened then.



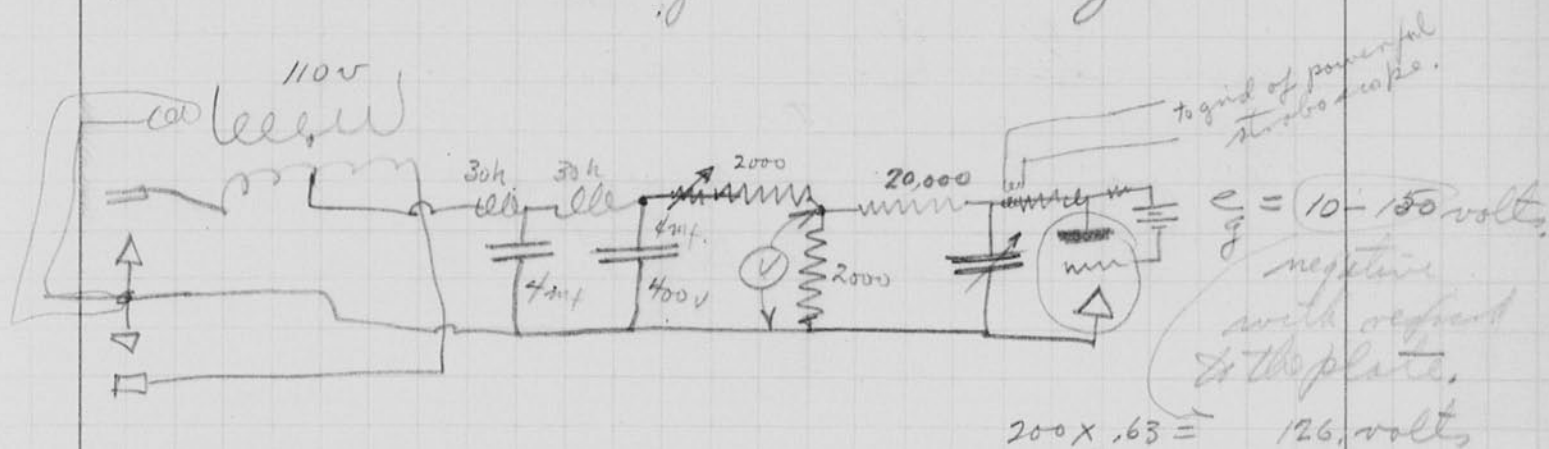
As shown in the figure the field first goes on, then if the motor fails to pull in and reaches 180° elect degrees the field will be taken off and the motor will start to gain its initial slip. However again at 0 degrees the field is connected to the exciter and the motor again tries to pull in from a smaller slip.

Thyratron Oscillator for driving stroboscope grids

May 3, 1931.
J. Edgerton



The frequency is determined by the R and the C of the charging circuit, ~~and~~ and by the bias voltage on the grid.



$$RC = \frac{1}{60 \text{ sec}} = 0.0167 = 20,000 C$$

$$C = \frac{.0167}{20,000} = \frac{1.67 \times 10^{-2}}{2 \times 10^4} = .8 \times 10^{-6} = .8 \text{ mf.}$$

Or make R = 100,000 ohms.

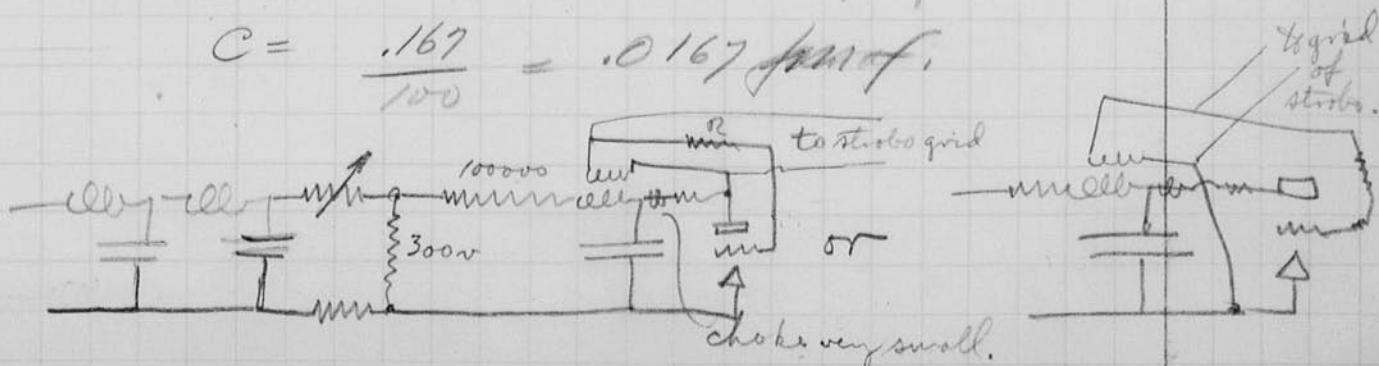
$$C = \frac{.0167 \times 10^{-2}}{10 \times 10^4} = .167 \times 10^{-6} = .167 \text{ mf.}$$

to make it oscillate 10 cycles/sec

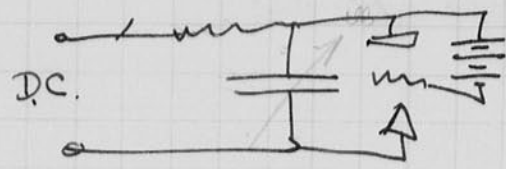
$$C = .167 \times 6 = 1002 \text{ mf.}$$

to make it oscillate 5000 cycles

$$C = \frac{.167}{100} = .0167 \text{ mf.}$$

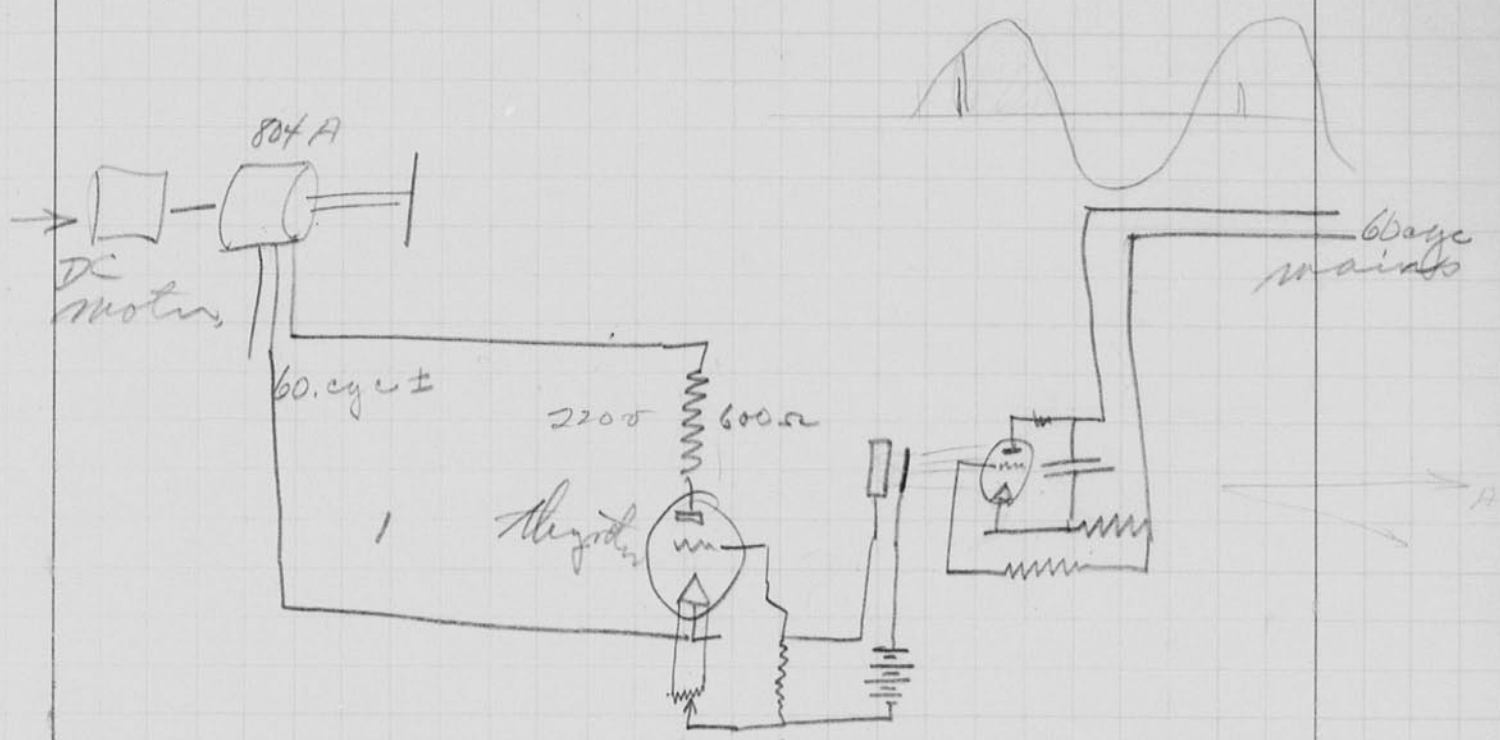


Not drawn
May 3, 1931
J. Edgerton



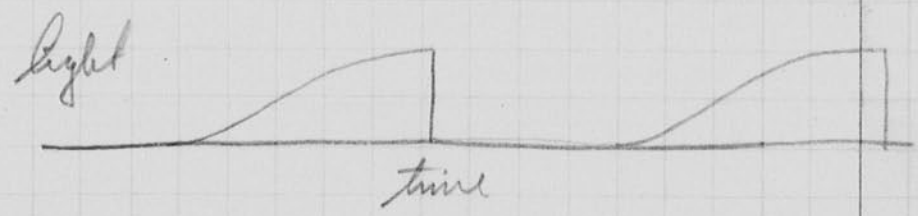
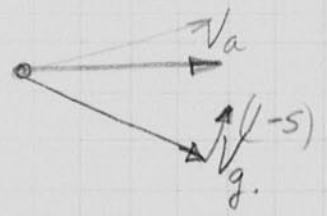
May 5/1931
 A. E. Egerton
 F. S. Gray.

Photocell operation with Stroboscopic Light.

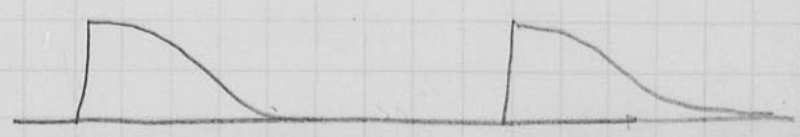


The light from the thyratron operated at a frequency which was the beat of the two frequencies

When the frequency of 804 A (alternator) was less than the mains the frequency light gradually increased to a maximum and then snapped off.



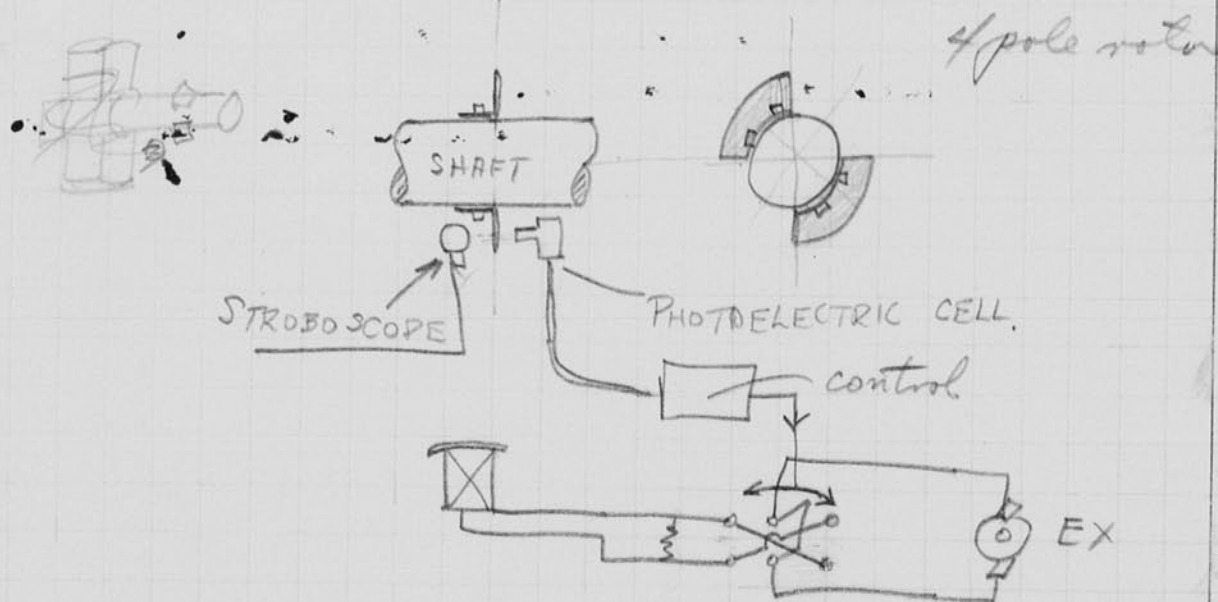
With the frequency of 804 A greater than the mains the variation of the light is the opposite



Field Switching Scheme.

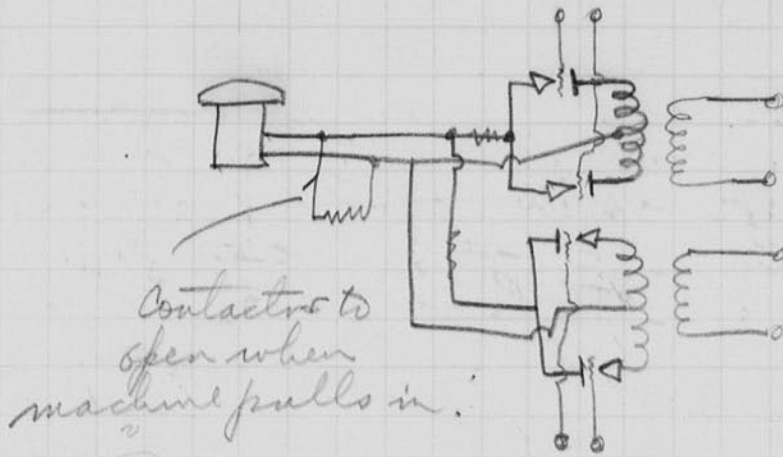
May 5/1931
W. Robertson

I propose to switch a synchronous motor field so that ~~it is always in~~ such the current in it is always in a sense as to give motor action. This can be accomplished by arranging a stroboscopic light opposite an obstruction driven by the motor which is ~~on half~~ of the time in the path of the stroboscopic light from ~~to~~ a photoelectric cell. The phase of the obstruction will be so arranged that it will allow the light to strike the photocell ^{only} when ^{stroboscopic} the angular displacement is between zero and 180 electrical degrees.



The field is connected one way when the stroboscopic light hits the ~~relay~~ photocell and is the other when the light is obstructed.

Cont.



Contactors to open when machine pulls in.

These can be one transformer.

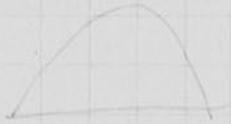
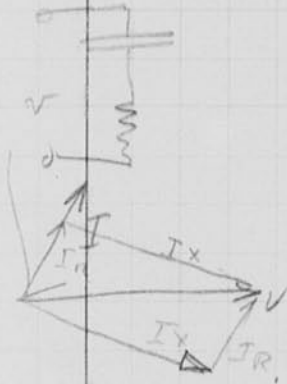


Photo cell and amplifiers are arranged to connect the proper rectifier to the field to make it give motor action.



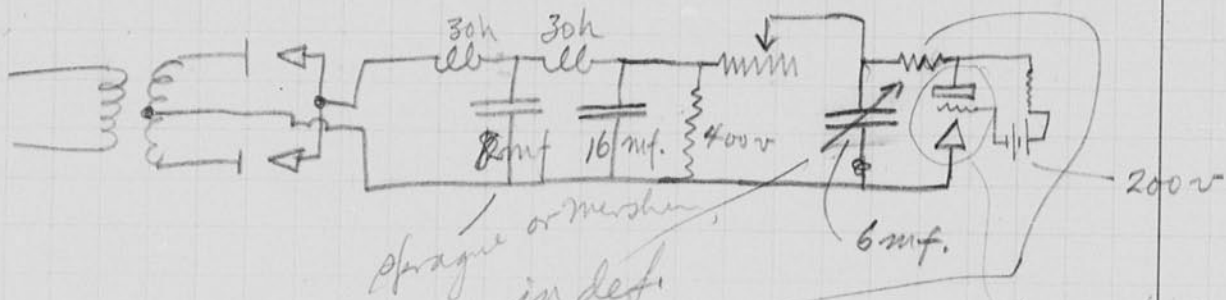
This circuit was shown to me on May 5, 1931 by H. E. Edgerton at Mass. Inst. of Technology, Cambridge, Mass.

Charles Kingsley Jr.

Witnessed May 13, 1931 E. L. Fowler

May 6, 1931.
H. J. ...

A-C operated variable frequency stroboscope.



Chaton grids
300 volts.

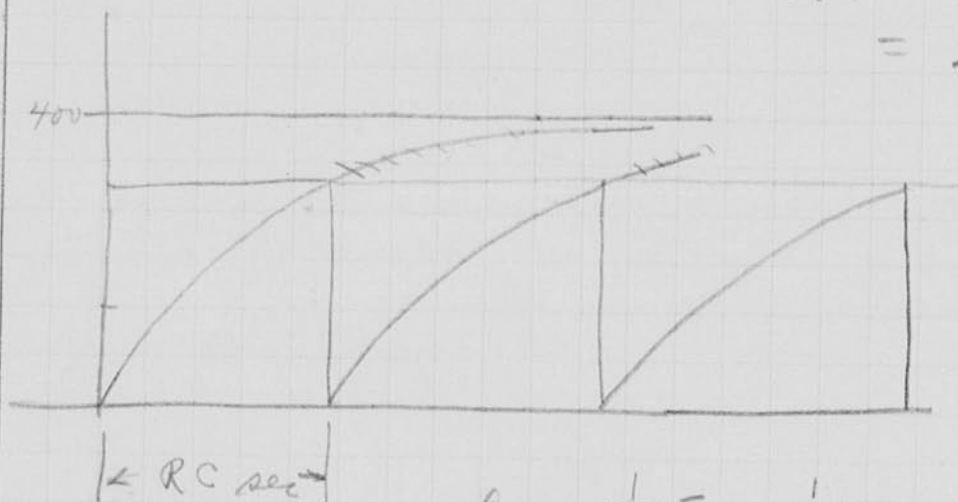
sprague or mercury
in def.
steps.

100 amp peak
thyristor.

$$\frac{400}{100} = 4 \text{ ohms.}$$

$$RC = 4 \times 6 \times 10^{-6}$$

$$= 24 \text{ ms.}$$



$$f = \frac{1}{RC} = \frac{1}{R \times 10^{-6} \times 6}$$

$$\frac{300}{1,000,000}$$

$$60 = \frac{1}{R \times 6 \times 10^{-6}}$$

$$R = \frac{60}{6 \times 10^{-6}} = 10 \times 10^6 \text{ ohms.}$$

$$RC = 6 \times 10^{-6} \times 10 \times 10^6 = 60 \text{ sec}$$

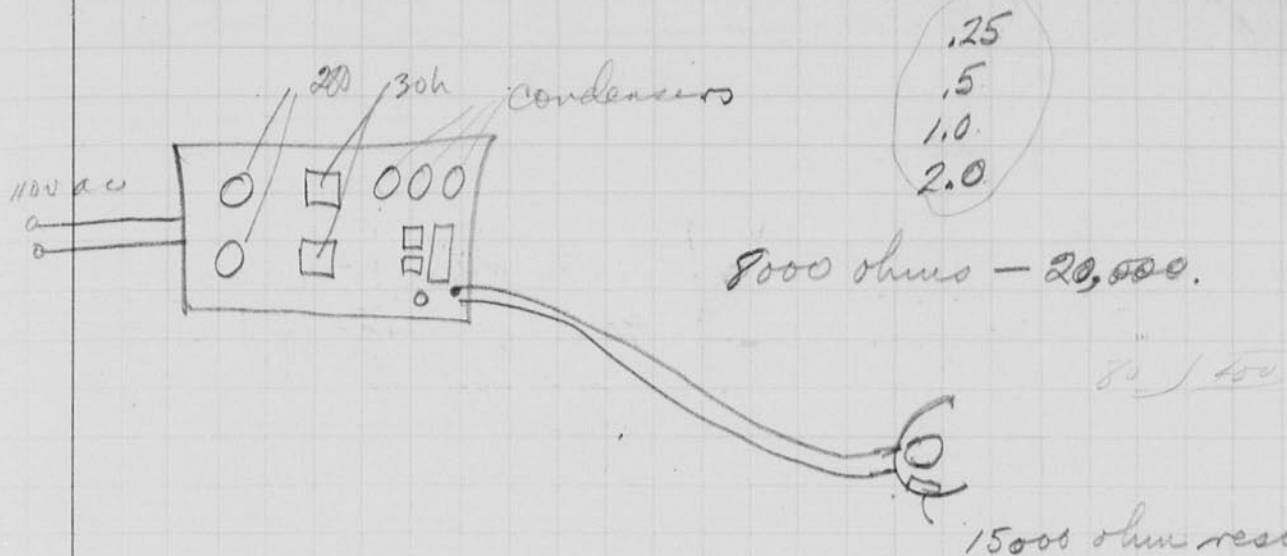
$$\frac{1}{60} = RC \quad R = \frac{1}{60 \times 6 \times 10^{-6}} = \frac{10^6}{360} = 3000 \text{ ohms.}$$

try 2 mf. for discharge and 4 ohms in disc. circuit

$$R = \frac{1}{60 \times 2 \times 10^{-6}} = \frac{10^6}{120} = 10,000 \text{ ohms.}$$

5,000 - 30,000 ohms. try amount
0.5 to 4 mf. condens.

Cont

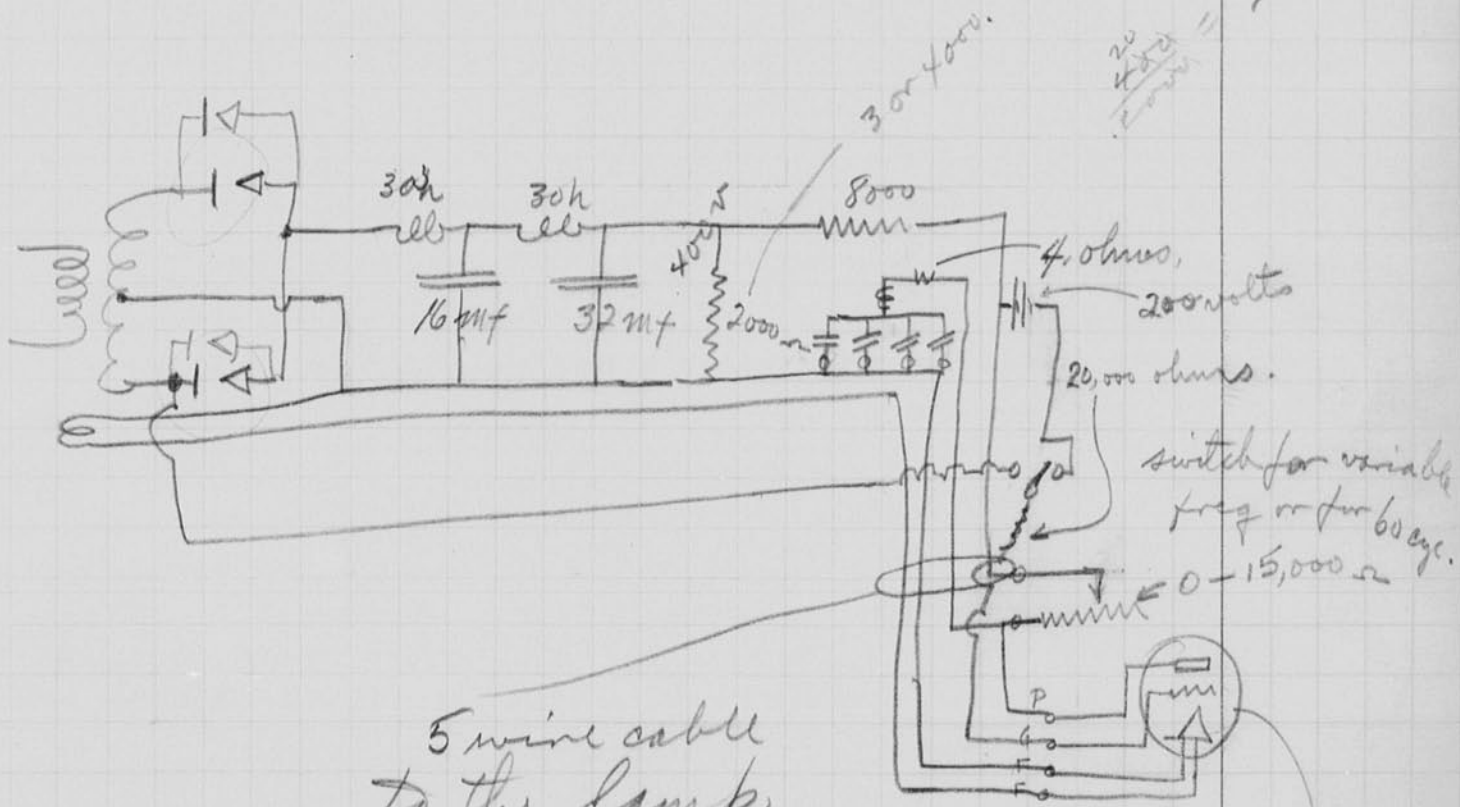


- .25
- .5
- 1.0
- 2.0

8000 ohms - 20,000.

15000 ohm resistance.

$$25 = \frac{50 \text{ ma. rating}}{2}$$



5 wire cable to the lamp.
The resistor in the hood gives freq. control right there where the light is.

$$.25 \times 10^{-6} \times 8000 = .002000 \times 10^{-6} = .002 \text{ sec.}$$

$$\frac{1}{.002} = 500 \text{ cycles. max freq. } 500 \times 60 = 30,000$$

$$3.75 \times 23,000 \times 10^{-6} = .090,000$$

$$\frac{1}{.09} = 11 \text{ cyc.}$$

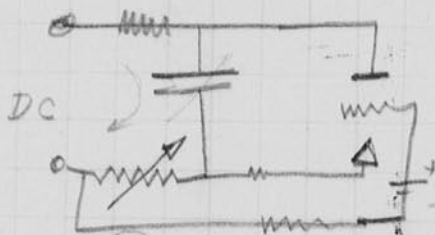
100 amp peak.

r.p.m.

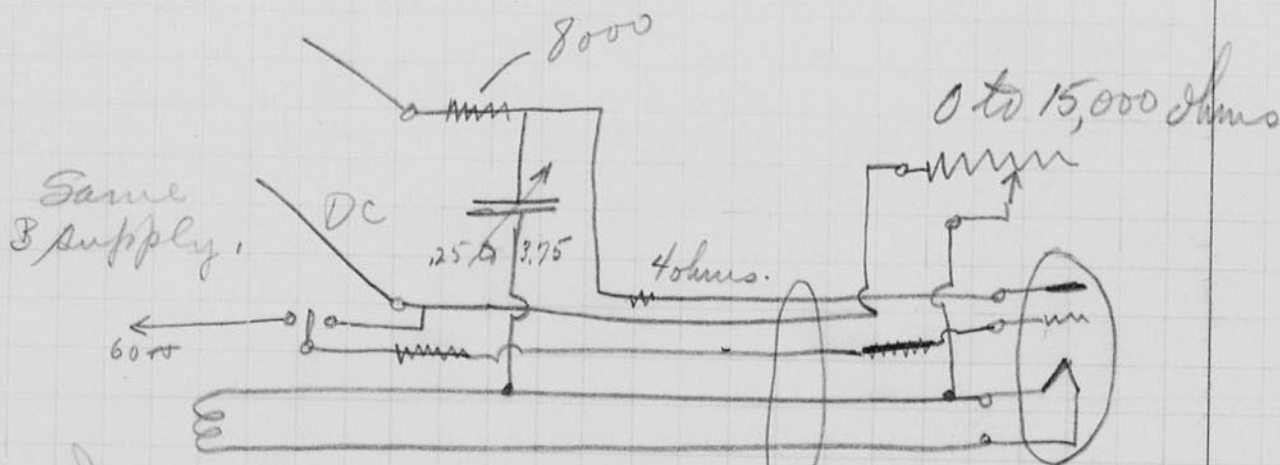
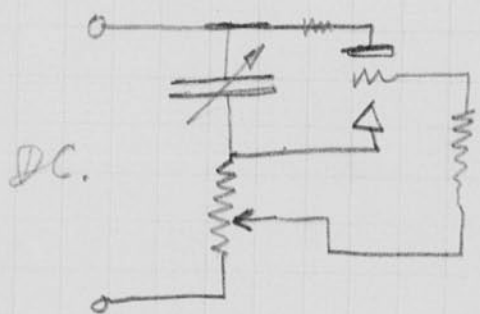
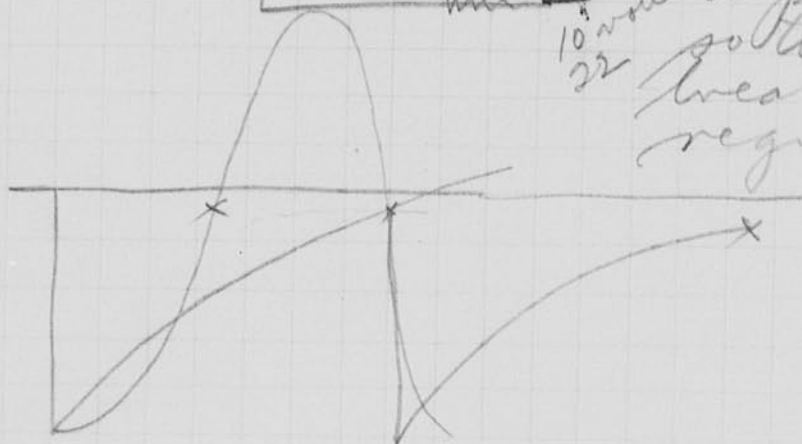
range

70
120
40

Cont,



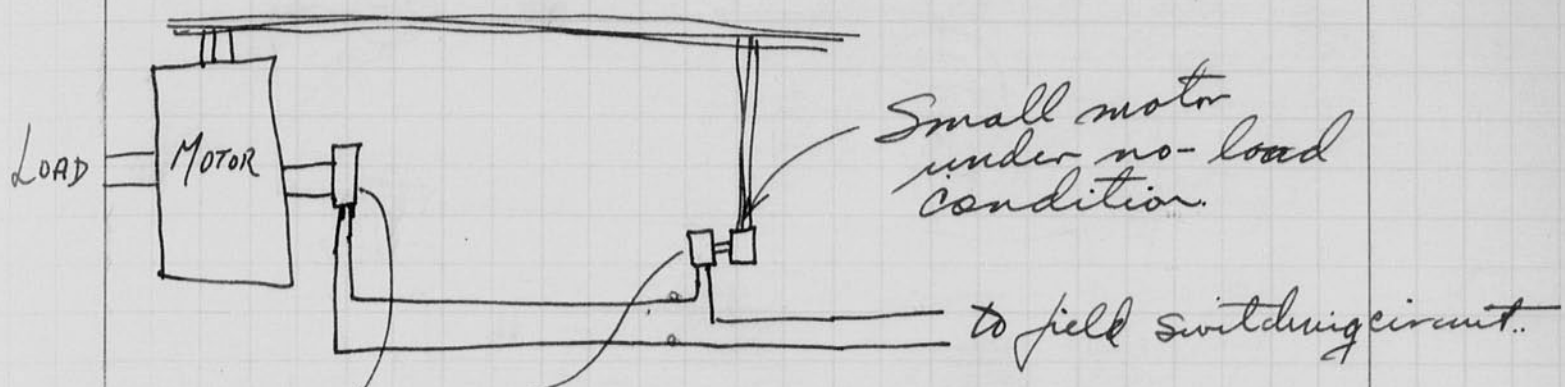
This eliminate the ~~c battery~~ large c battery but will require a positive c bias so the tube will break down regularly.



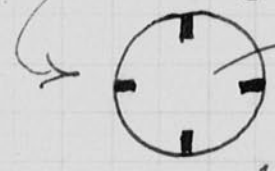
W. H. ...
T. ...
May 1931
L. ...
May 1931

5 conductor cable

May 14 1931 Field Switching Scheme for a
H.E. Edgerton Synchronous Motor.



Commutator to have one ~~one~~ segment for each pair of poles.



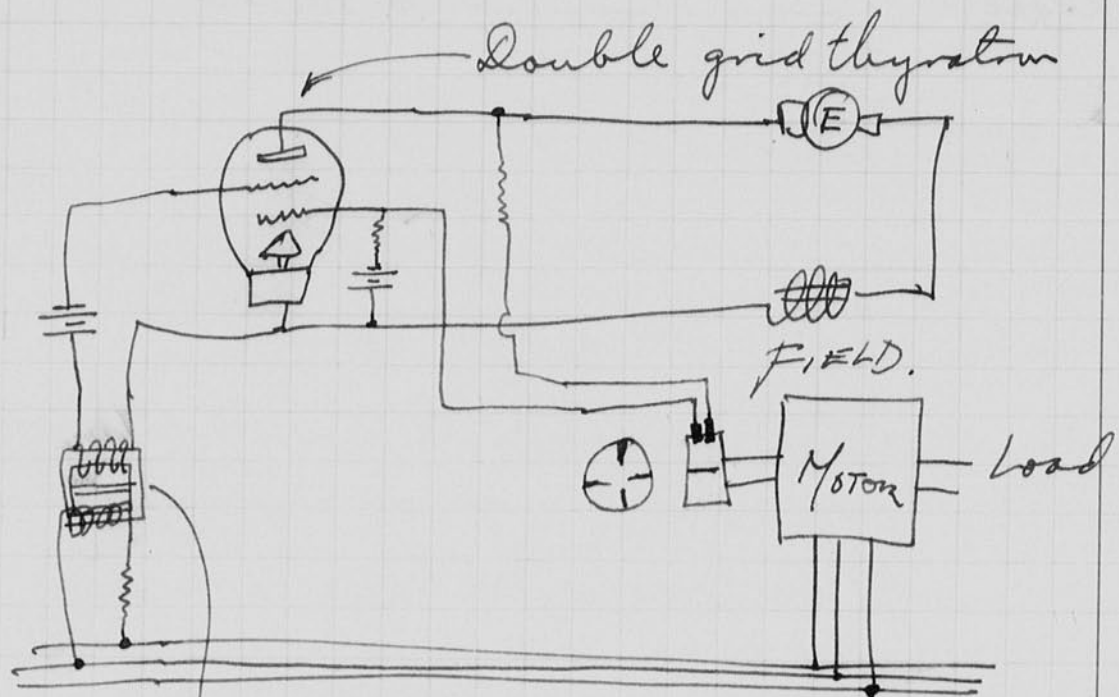
insulation.

The electrical circuit through the field switching arrangement is only made when the two ~~commutators~~ ^{gets at full load} are ~~on their~~ both on their respective commutator segments at the same time.

One disadvantage of this scheme is that the electrical contact is not made very long and so the ~~impulse~~ ^{impulse} to the trip circuit needs to be amplified by some sort of ^{sensitive} relay.

An advantage of this scheme is that the small synchronous motor ~~need only to~~ does not need to be lined up with the big motor and can be placed anywhere on the control board etc.

Cont. Field Switching Schemes.



Saturated core transformer
to give a peaked voltage.

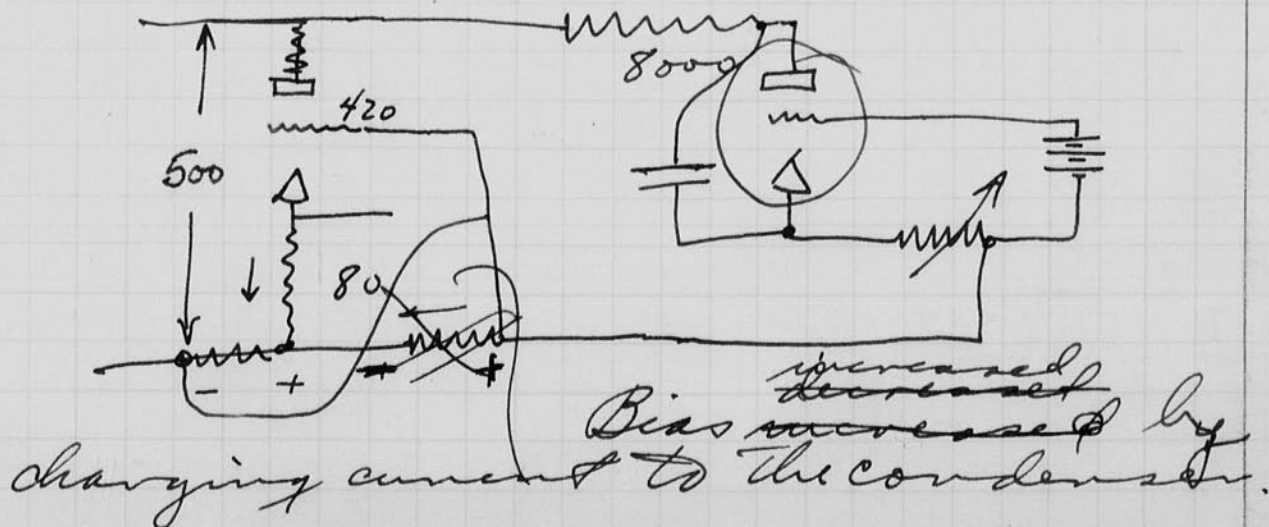
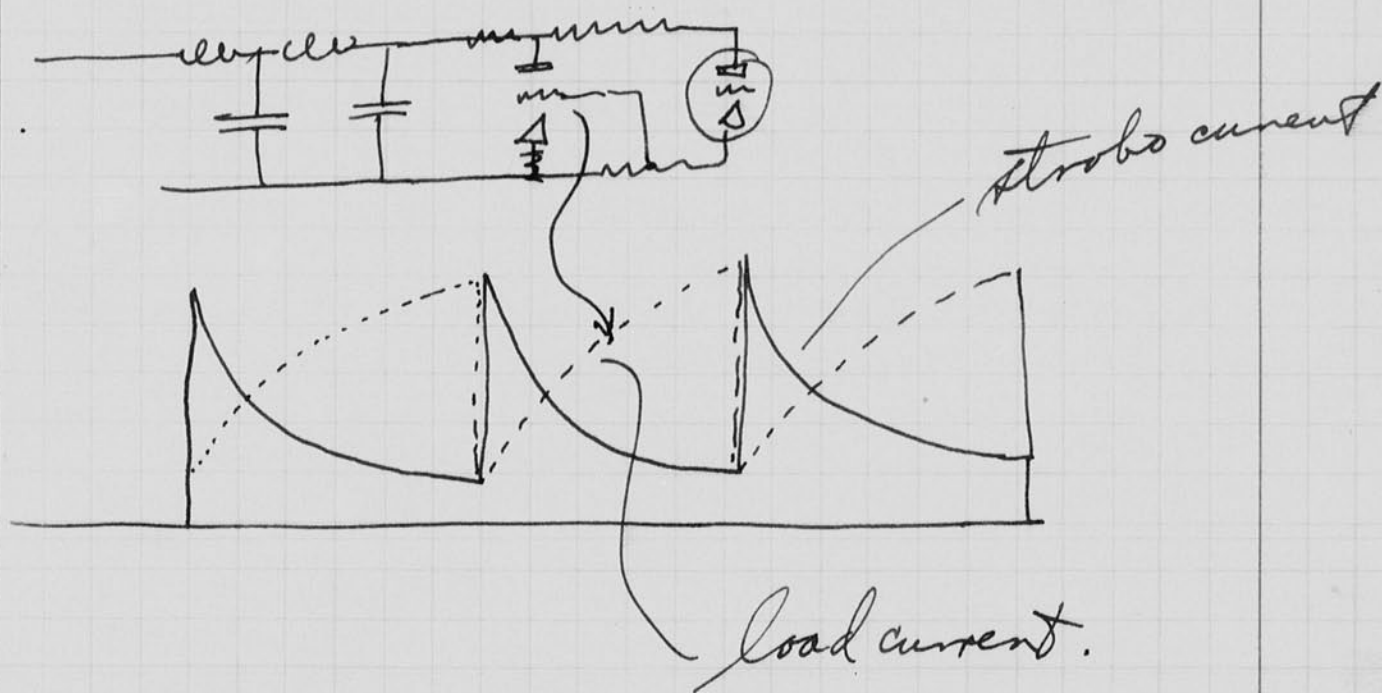
The thyristor will not conduct until the two grids are positive. This only occurs when the angular displacement has a certain phase relationship ~~to the~~ which is adjusted so that it occurs at the most favorable switching angle, that is, zero degrees.

It would be more accurate to say that the thyristor does not conduct until there is a certain relationship between the potentials on the two grids. If the necessary voltages to cause breakdown may be either positive or slightly negative.

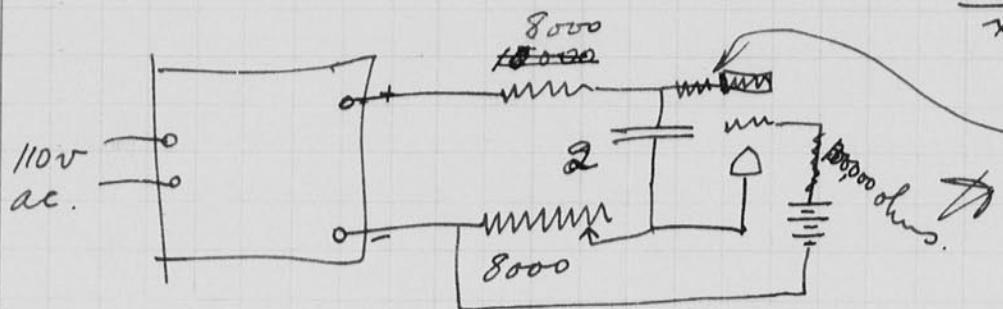
May 16 1931
 H. Edgerton

Constancy of the frequency of the stroboscope shown on pages 88-90 depend upon a constant d.c. voltage. One method to get this is to have a large enough "b" eliminator so that the load current does not appreciably change the voltage.

A load just like the stroboscope but arranged so that the sum of the current is constant may help to keep the frequency constant.



Cont. Scheme to be tried.



$$\frac{500}{x} = 25 \quad 4 \times 5$$

$$x = \frac{200}{25}$$

22V. = 50 ohms.
limit current.

$$RC = \frac{1}{60} \text{ sec.}$$

$$C = 2 \times 10^{-6}$$

$$R = \frac{10^6}{60 \times 2} = \frac{10^4}{1.2 \times 10^2}$$

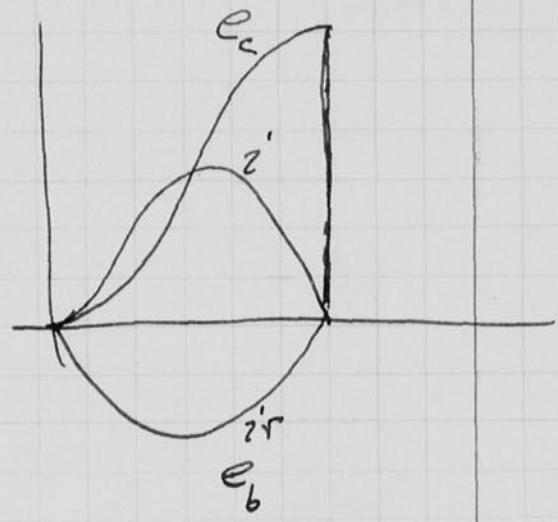
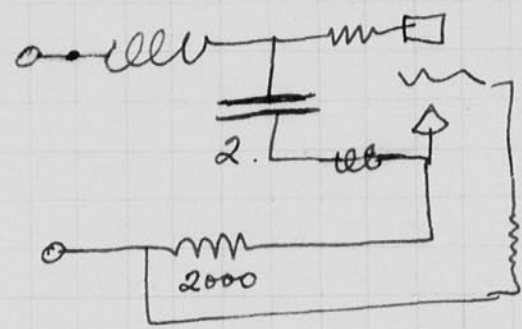
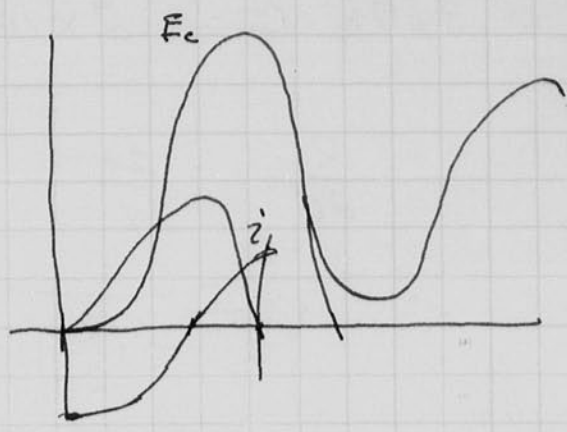
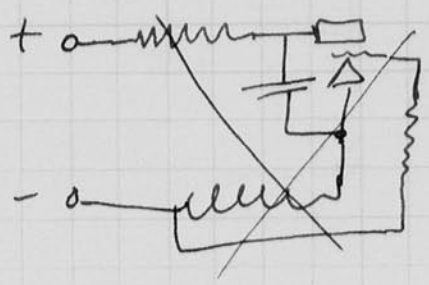
$$= 10,000 \text{ ohms.}$$

Material list for strobe.

- 2 mf good for 100v. d.c. .25 to 2 mf steps.
- ✓ 1 - 8000 ohms
- ✓ 1 - 8000 ohm slide.
- 1 - 100,000 ohms
- 1 - "B" battery 22 volts.
- 1 - 50 ohm slide wire.

F4-27 and fil. transformer.

Cont.



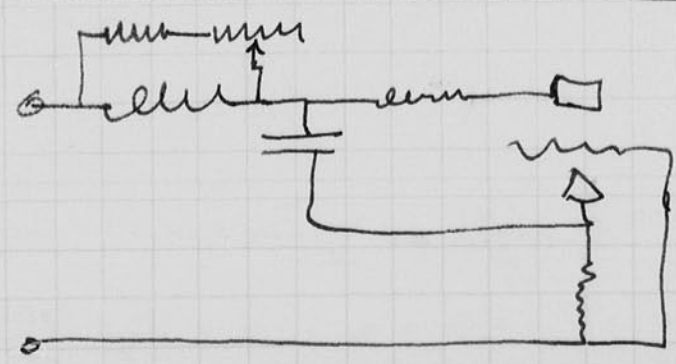
$$\frac{1}{2\pi\sqrt{LC}} = f$$

$$T = 2\pi\sqrt{LC}$$

$$\frac{1}{60} = 2\pi\sqrt{2 \times L}$$

$$\frac{1}{3600} = 4\pi^2 \times 2 \times L$$

$$L = \frac{1}{36,000 \times 4\pi^2 \times 2 \times 10^6 \times 288 \times 10^2} = \frac{1}{288,000} = \frac{1}{288} = .347 \mu\text{h}$$



$$X = \frac{R}{2L} \quad \frac{L}{R} = \frac{.347}{2000}$$

$$X = \frac{2000}{2 \times .347} = \frac{3000}{.694}$$

$$L = .347 \text{ h} \quad C = 2 \times 10^{-6} \text{ f} \quad R = 2000 \text{ } \Omega$$

$$\frac{R}{2L} = \frac{2000}{2 \times .347} \approx 3000$$

$$\frac{R}{\sqrt{\frac{L}{C}}} < .268$$

$$\sqrt{\frac{L}{LC - \left(\frac{R}{2L}\right)^2}} = \sqrt{\frac{1.43 \times 10^6}{10 \times 10^6}}$$

$$\frac{1}{LC} = \frac{1}{.347} \frac{1}{2 \times 10^{-6}} = \frac{10^6}{.7} = 1.43 \times 10^6$$

$$\frac{3000}{10,000,000}$$

$$\frac{3000}{10 \times 10^6}$$

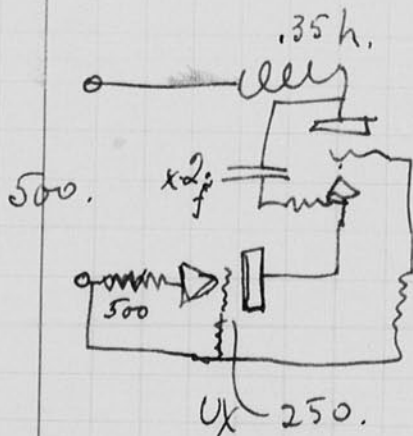
the circuit is over damped.

R must be less than

$$\left(\frac{R}{2L}\right)^2 < 1.43 \times 10^6$$

$$\frac{R}{2L} < 1.2 \times 10^3 = 1200$$

$$R = 2400 \times .35 = \underline{840 \text{ } \Omega}$$



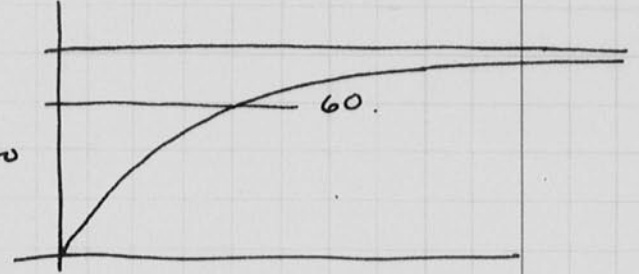
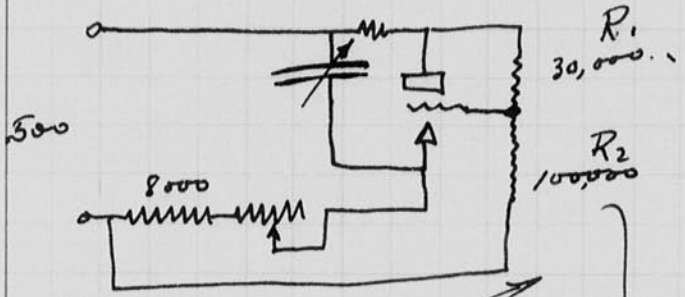
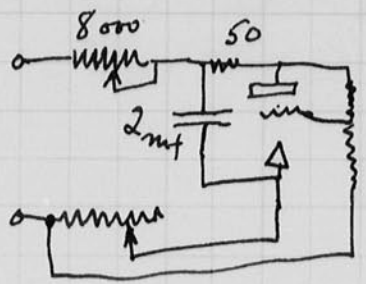
too complicated?!

$$10 \text{ mf} \times 20 = 200 \text{ m.f.}$$

$$\frac{0.800}{2} = 2.$$

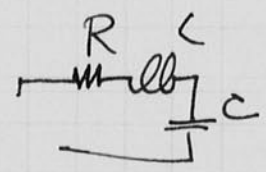
$$\sqrt{.35} = .41$$

May 10 1936
H. C. Roberts



make $\frac{R_2}{R_1} = 3$ approx.

This makes the grid go positive before the con. as soon as the condenser has built up to 60% of its final value it would have if the tube did not strike.



$$L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{1}{C} q = \mathcal{E} = E_1$$

Let $t = a\lambda$

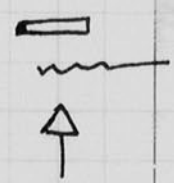
$$a^2 \frac{d^2q}{d\lambda^2} + \frac{aR}{L} \frac{dq}{d\lambda} + \frac{a^2}{LC} q = a^2 \frac{E}{L} 1$$

$$\left(\rho^2 + a \frac{R}{L} \rho + \frac{a^2}{LC} \right) q = a^2 \frac{E}{L} 1 \quad \rho = \frac{d}{d\lambda}$$

Let $\frac{a^2}{LC} = 1 \quad a = \sqrt{LC}$

$$\rho^2 + \frac{R\sqrt{LC}}{L} \rho + 1 = \frac{LC E}{L} 1$$

$$\left(\rho^2 + \frac{R}{\sqrt{LC}} \rho + 1 \right) q = EC 1$$



$$\left(p^2 + \frac{R}{\sqrt{L/C}} p + 1\right) = 0 \quad \text{for roots.}$$

$$p = \frac{-R}{2\sqrt{L/C}} \pm j \sqrt{1 - \left(\frac{R}{2\sqrt{L/C}}\right)^2}$$

critically damped case

$$\frac{R}{2\sqrt{L/C}} = 1.$$

$$R = 2\sqrt{L/C} \quad \sqrt{.17} = .41$$

$$= 2\sqrt{\frac{.35}{2 \times 10^{-6}}}$$

$$= 2 \times 10^3 \times \sqrt{\frac{.35}{2}} = 800 \text{ ohms.} = 8 \times 10^3$$

In a similar manner the current instead of the charge.

$$\left(\frac{di}{dt} + R + \frac{1}{C} \int dt\right) i = E \mathbb{1}.$$

$$\text{Let } t = a\lambda.$$

$$\left(\frac{dL}{a d\lambda} + R + \frac{a}{C} \int d\lambda\right) i = E \mathbb{1}.$$

$$\left(\frac{d}{d\lambda} + \frac{Ra}{L} + \frac{a^2}{Lc} \int d\lambda\right) i = a \frac{E}{L} \mathbb{1}.$$

$$a^2 = Lc. \quad p = \frac{d}{d\lambda}.$$

$$\left(p + \frac{R}{\sqrt{L/C}} + \frac{1}{p}\right) i = \frac{E}{\sqrt{L/C}} \mathbb{1}.$$

$$\left(p^2 + p \frac{R}{\sqrt{L/C}} + 1\right) i = p \frac{E}{\sqrt{L/C}} \mathbb{1}.$$

$$i = \frac{p E / \sqrt{L/C} \mathbb{1}}{\left(p^2 + p \frac{R}{\sqrt{L/C}} + 1\right)}$$

$$\frac{P}{(\rho^2 + 2\alpha\rho + \omega_0^2)} = \frac{E}{\omega} \sin \omega t + 1$$

$$\omega_0^2 > \alpha^2 = \omega_0^2 - \alpha^2$$

$$\tan \phi = \omega / \alpha$$

~~$$\alpha = \frac{R}{2L}$$~~

$$\alpha = \frac{R}{2\sqrt{LC}}$$

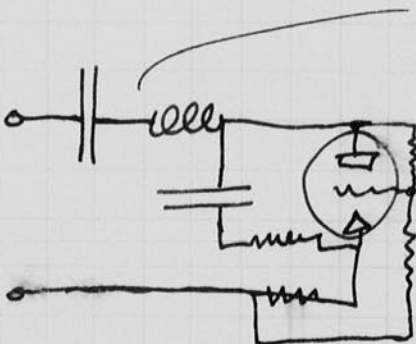
$$i = \frac{E}{\sqrt{\frac{L}{C}}} \frac{e^{-\alpha t}}{\omega} \sin(\omega t + 1)$$

$$i_{max} = 2 E \sqrt{\frac{C}{L}} \text{ for } \alpha = 0 \text{ no damping.}$$

$$L = .35 \quad C = 2 \times 10^{-6} \quad E = 500$$

$$i_{max} = 2 \times 500 \sqrt{\frac{2 \times 10^{-6}}{.35}} = 1000 \times 10^{-3} \sqrt{5.7} = 2.4 \text{ amperes.}$$

to prevent shorting the tube if the grid fails.



$$\frac{L i^2}{2} = \frac{1225}{2} = 112 \text{ joules.}$$

$$\frac{C E^2}{2} = 112$$

$$E = \frac{225}{10 \times 10^{-6}} = 200 \times 10^6 \text{ volt}$$

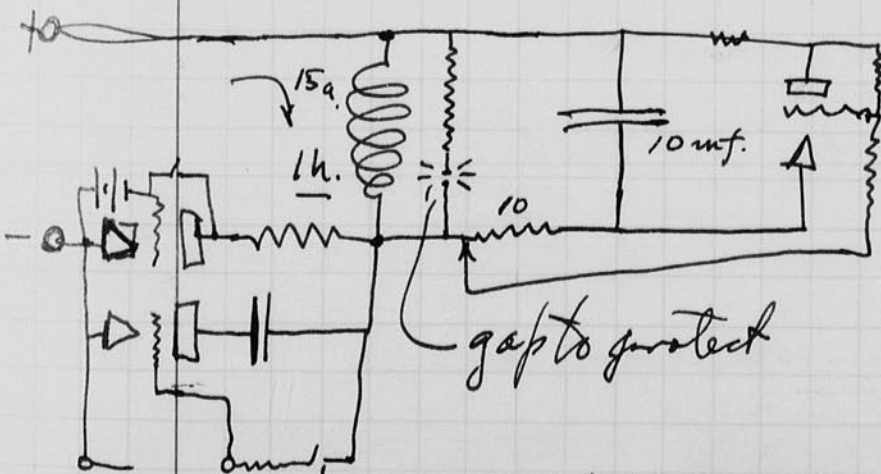
if tube has max capacity 10,000 cycles per sec.

$$\frac{1}{10000} \text{ sec per cycle.}$$

$$1 \times 10^{-4} \text{ sec.} = \text{time const}$$

$$1 \times 10^{-4} = 10 \times 10^{-6} R.$$

$$R = \frac{1 \times 10^{-4}}{10 \times 10^{-6}} = \frac{10^2}{10} = 10 \text{ ohms.}$$

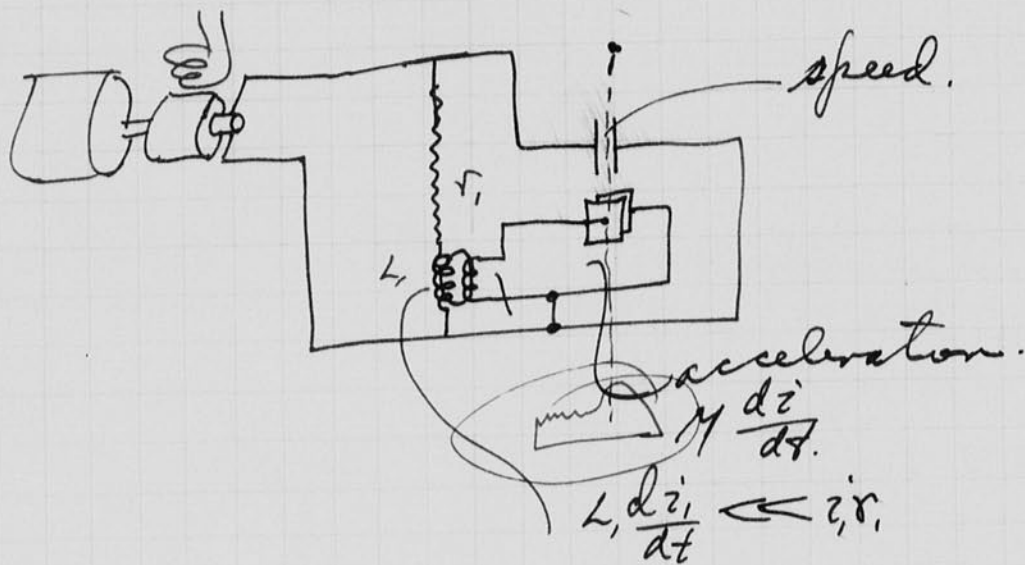


switch to start transients.

if tube has max capacity 10,000 cycles per sec.

11 May 17 1931
H. S. Edwards.

Speed - torque (acceleration)
measurement by means
of the cathode ray oscillograph.



∩ Ripple must be less than 0.5%
 $M \frac{di}{dt}$ must give voltage enough
to deflect beam.

say $\frac{.5 \text{ amp.}}{.1 \text{ sec}} = 5 \text{ am./sec} = \frac{di'}{dt}$

$100 \text{ volts} = M \frac{di'}{dt}$

$M = \frac{100}{5} = 20 \text{ h.}$

$\frac{L_2}{L_1} = \left(\frac{N_2}{N_1}\right)^2$

$\frac{100}{.5} = 200 \text{ ohms.}$

$L \frac{di'}{dt} \ll 100$ say 1 volt.

$L = \frac{1}{5} = 0.2 \text{ h.}$

$M = \sqrt{L_1 L_2}$

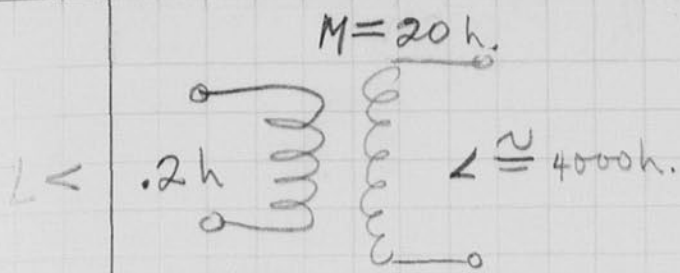
$L_2 = \frac{M^2}{L_1} = \frac{20 \cdot 20}{0.2} = \frac{4000}{.2} = 20,000 \text{ h.}$

This transformer needs a very
large ratio of turns.

$\left(\frac{N_2}{N_1}\right)^2 = \frac{4000}{0.2} = 20,000$

$\left(\frac{N_2}{N_1}\right) = 140.7 \text{ ratio.}$

50 turns.
7500 turns.



Ratio of turns
150 to 1.

$L \frac{di}{dt} = j\omega L i$ for ac:
 at 60 cycles. $377L \ll 200 \text{ ohms}$.
 say 2 ohms.

$$L = \frac{2}{377} = 0.0053 h.$$

if M needs to be from 15 to 100 h.

$$\left(\frac{N_2}{N_1}\right)^2 = \left(\frac{L_2}{L_1}\right) \quad M = \sqrt{L_1 L_2} \text{ unity coupling.}$$

$$L_1 = \frac{M^2}{L_2}$$

$$\frac{N_2}{N_1} = \sqrt{\frac{L_2 L_2}{M^2}} = \sqrt{\frac{L_2^2}{M^2}} = \frac{L_2}{M} \quad L_2 = \frac{M^2}{L_1}$$

$$\frac{N_2}{N_1} = \sqrt{\frac{L_2}{L_1}} = \sqrt{\frac{M^2}{L_1^2}} = \frac{M}{L_1}$$

Let $M = 15 \quad L = 0.053.$

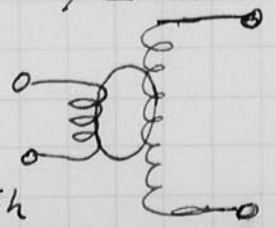
$$\frac{N_2}{N_1} = \frac{15 \times 377}{2} = 2800 \text{ to } 1.$$

$M = 100 \quad L = 0.053.$

$$\frac{N_2}{N_1} = \frac{100 \times 377}{2} = 18,600 \text{ to } 1.$$

h

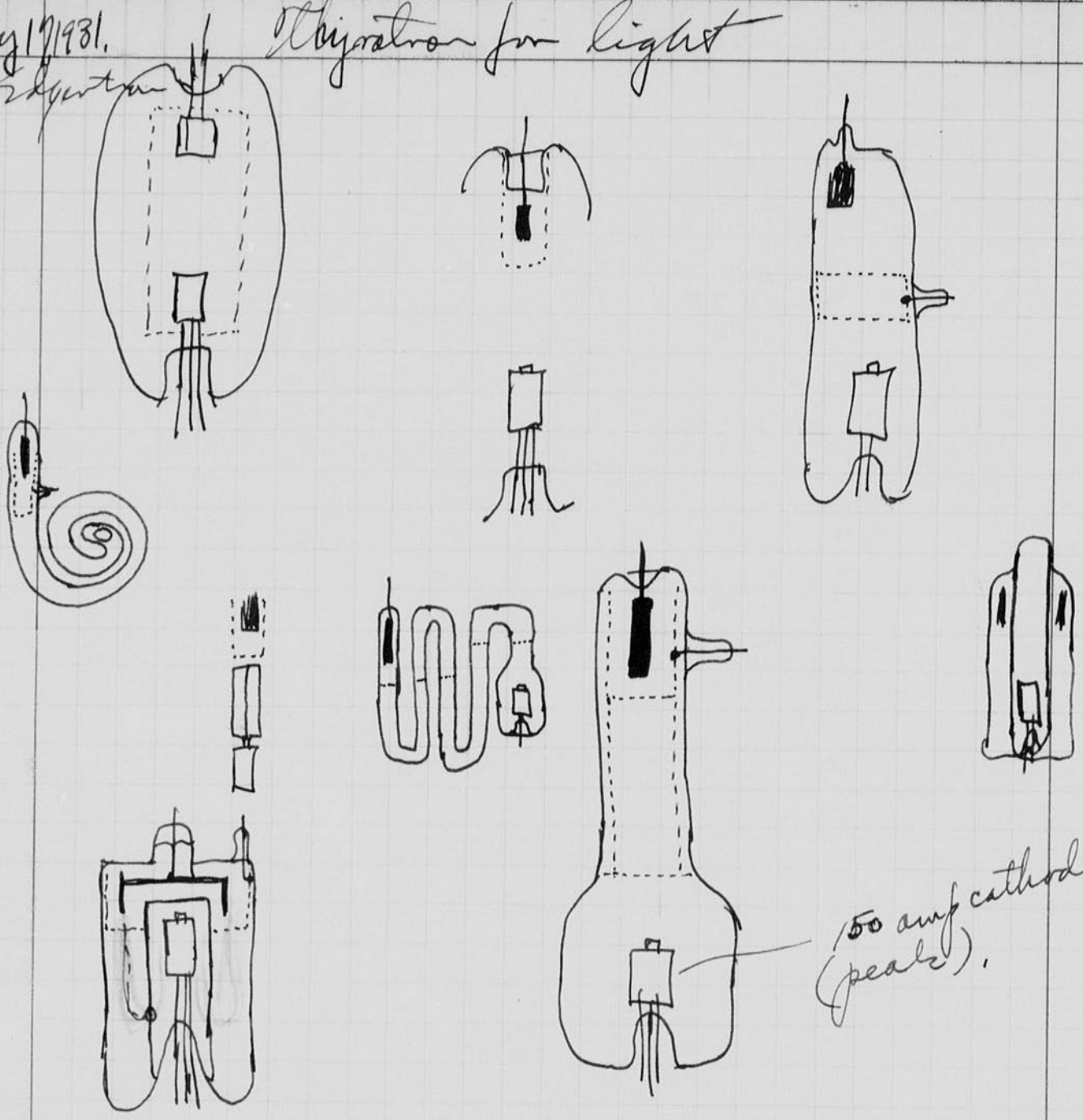
- / - 2000
- / - 20000



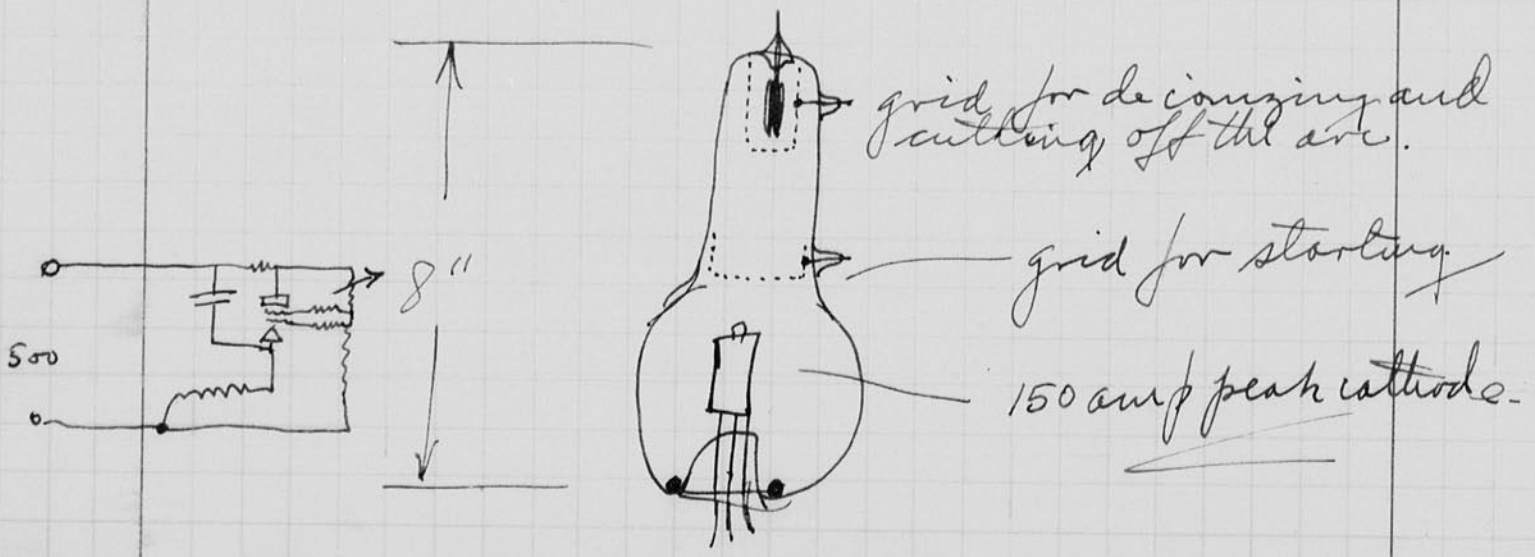
(0.5 amp dc.) no non-linear effects.

May 17 1931.
A. G. Edgerton

Thyratron for light

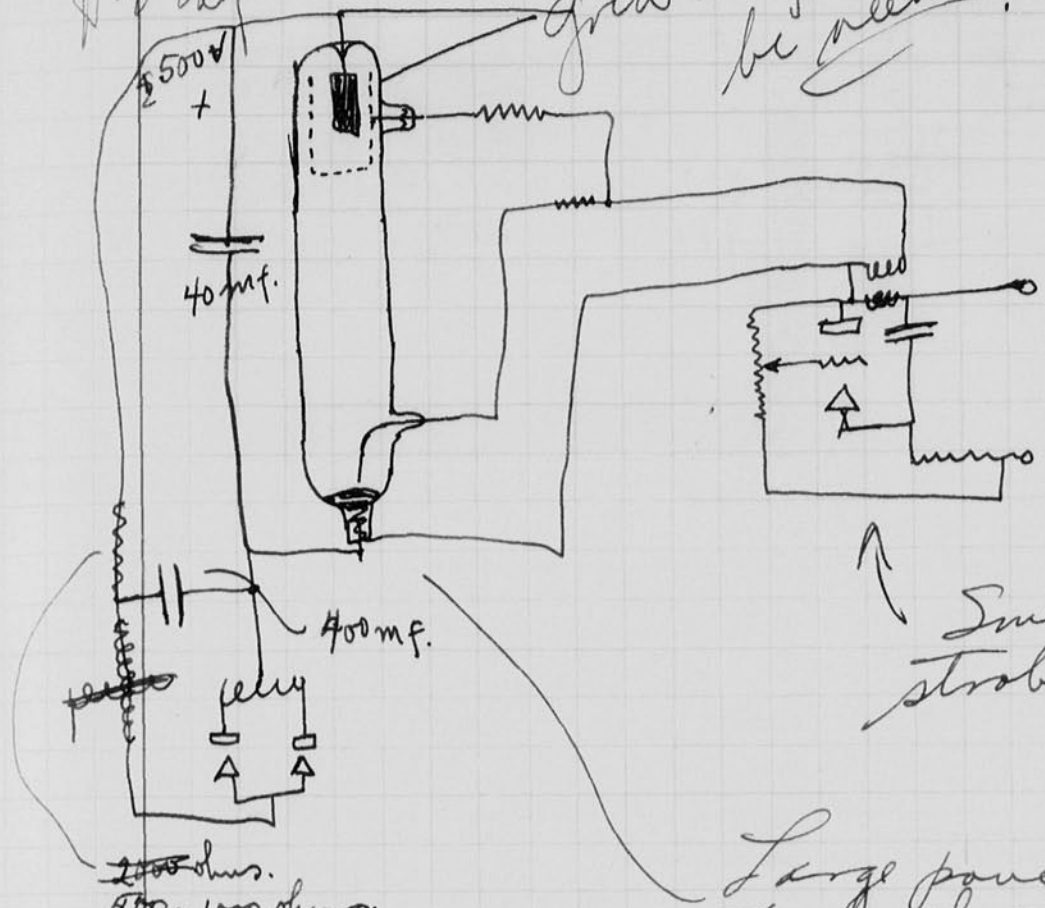


50 amp cathode (peak).



May 17 1937
H. S. G. S. S.

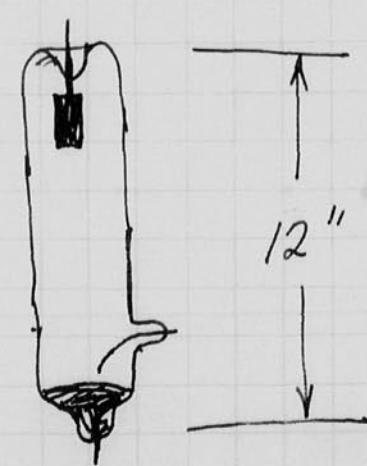
grid may not be needed.



Small visual stroboscope.

Large powerful strobo to be tripped by the small one.

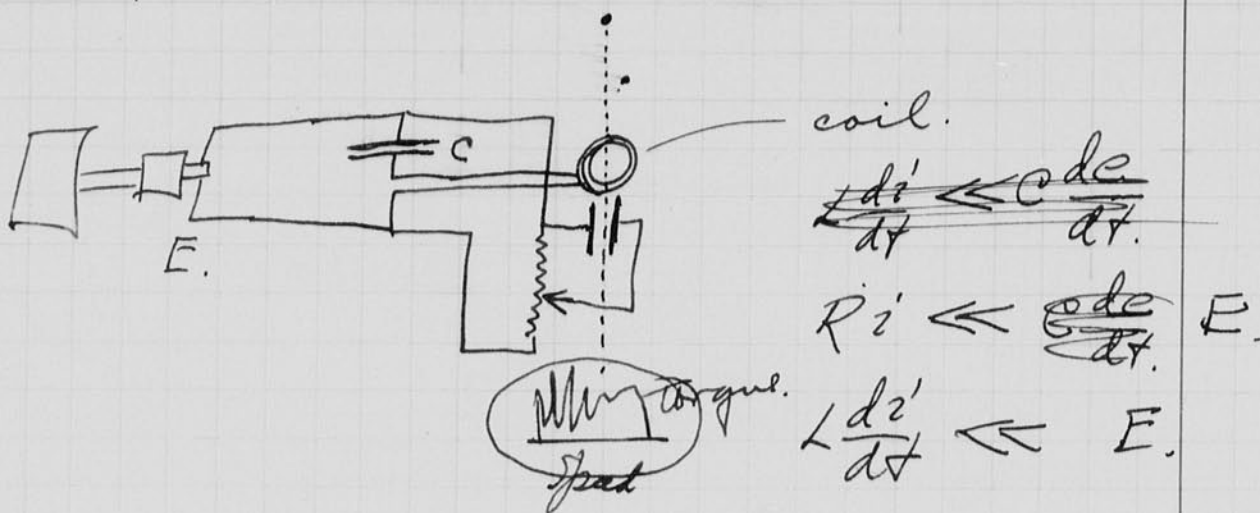
No holding spot needed. The discharge of the small strobo is put through a spark coil which forms a spot on the big tube's pool.



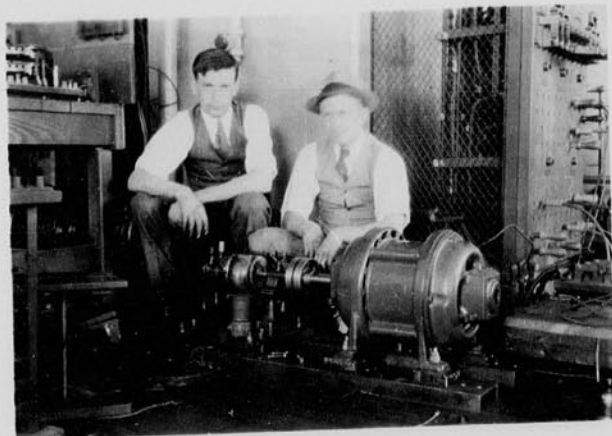
Wrap externally with wire and connect it to the starting anode.

May 18, 1931.
 Speed-torque

Speed-torque Cathode ray tests.



Jim Byrne



Dick Mason.

with pulsationless-free generator

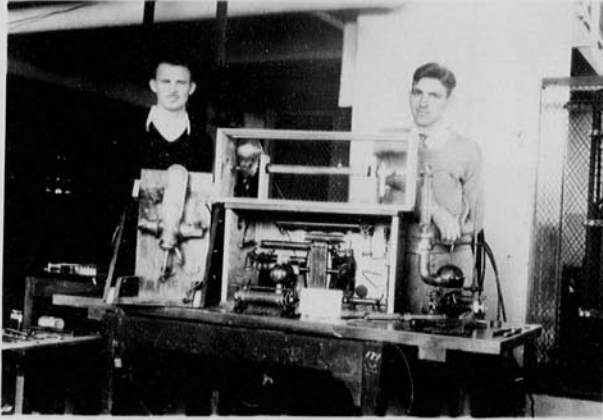
May 19 1931

J.B. McClure

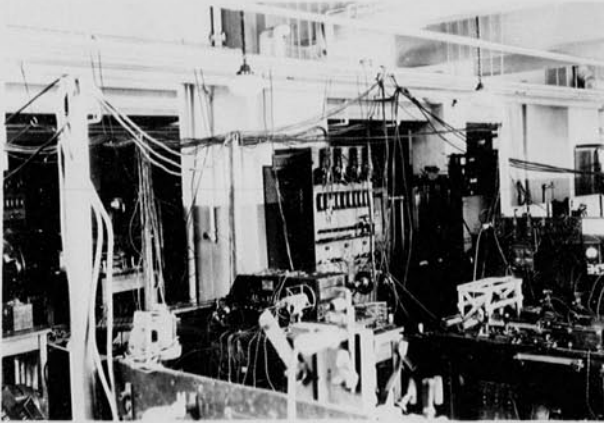
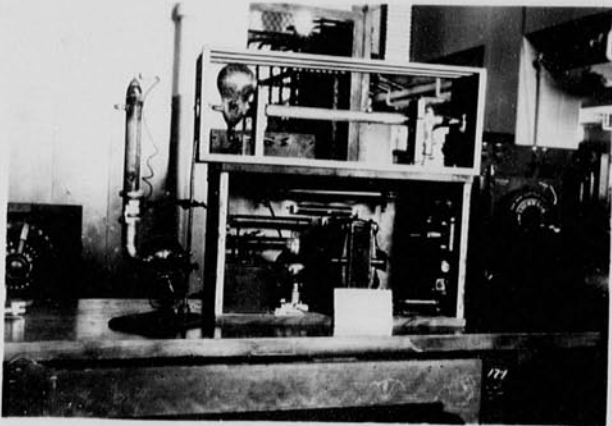
Photographs taken

May 19, 1931.

Lawrence
M. Stauder

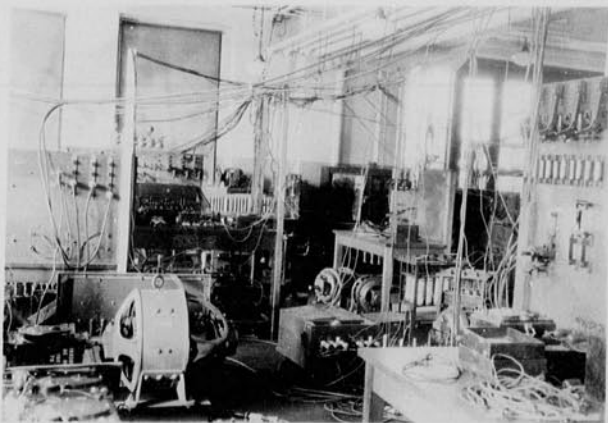


Stroboscope equipment



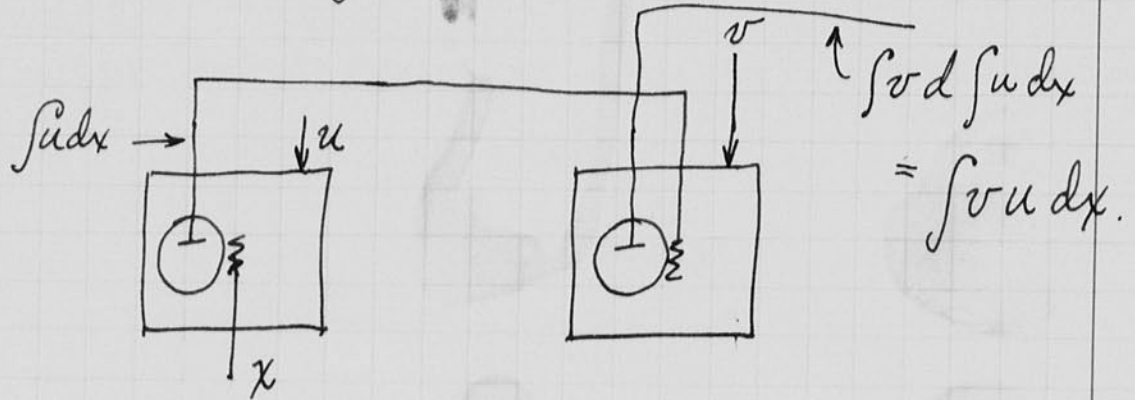
Machine transients
Laboratory.

Spring 1931.

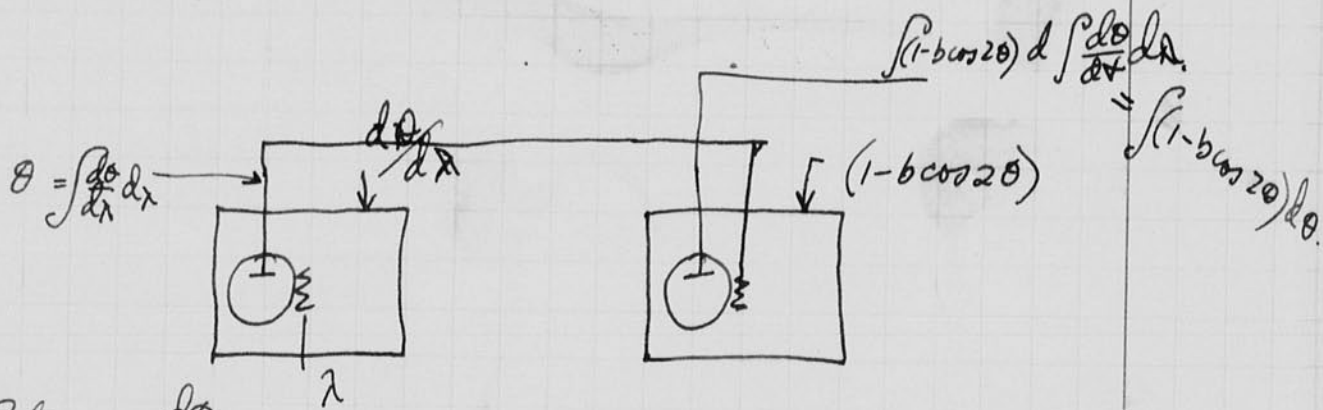


Method of Getting $\int u v dx$ on the Integragraph.

$\int u v dx$.



In the ~~synchronous~~ synchronous machine problem the term $\int (1 - b \cos 2\theta) \frac{d\theta}{dt} dt$ can be solved in this manner.



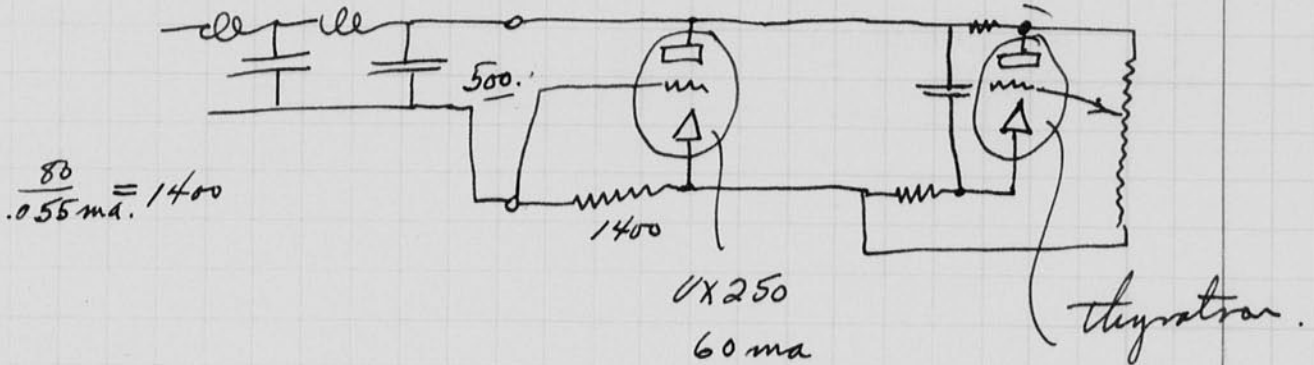
Let $u = \frac{d\theta}{dt}$.

$v = (1 - b \cos 2\theta)$.

May 22/93/
A.L. Edgerton.

Cont of strobo of page 93.

V.T. adjusted so that it will draw a current such that the current from the filter is constant.



$$\frac{80}{0.55 \text{ mA}} = 1400$$

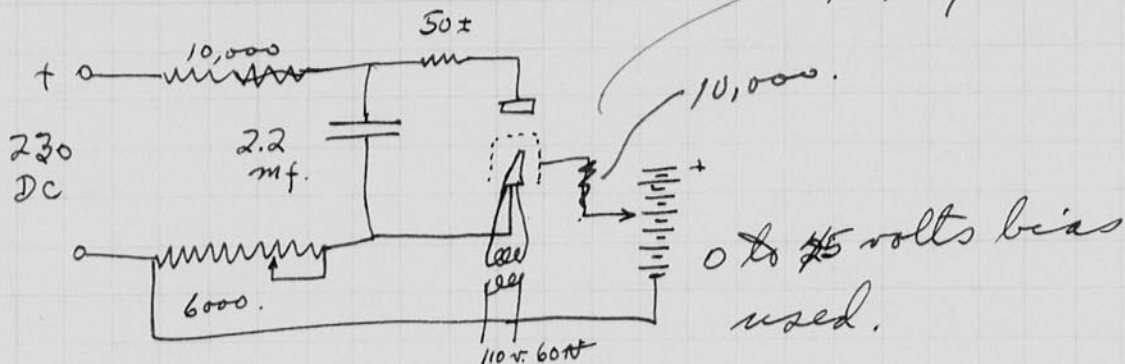
Design Resistances etc ~~to~~ so that the total current is a constant. The charging current increases the bias and reduces the current drawn by the 250. tube.

May 26, 1931.

Stroboscope.

Circuit described on page 94 connected up and tried.

F.G. 27.

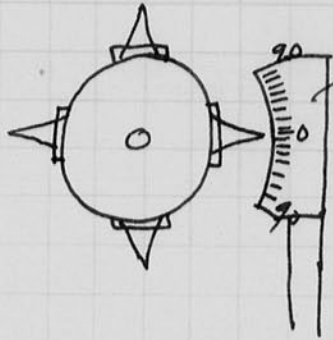


There is a periodic variation of the flashes which is apparently due to the interference of the voltage on the filament. The 60 cycles beat with the frequency of the flash circuit.

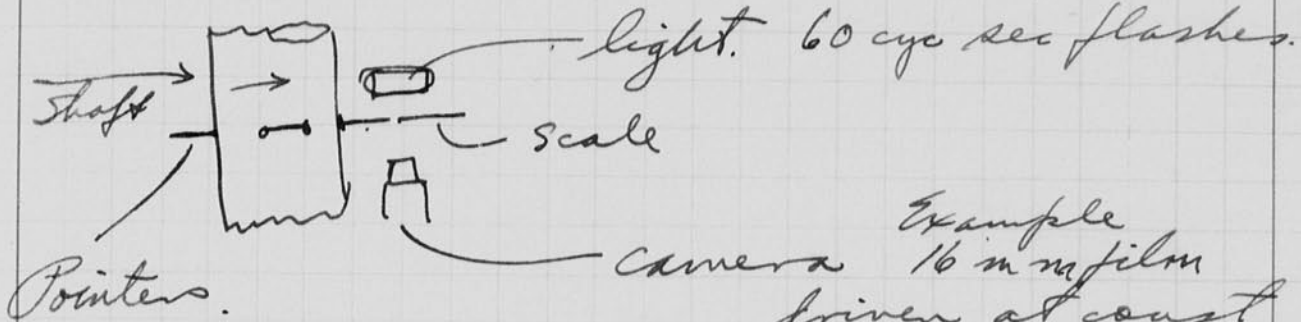
As F.G. 33 did not work on this circuit for some reason. It would flash when the grid was connected to the anode but would not operate with the grid + ~~25~~ 45 volts!

Stroboscopic Movies.

May 28, 1931
A. S. Edgerton



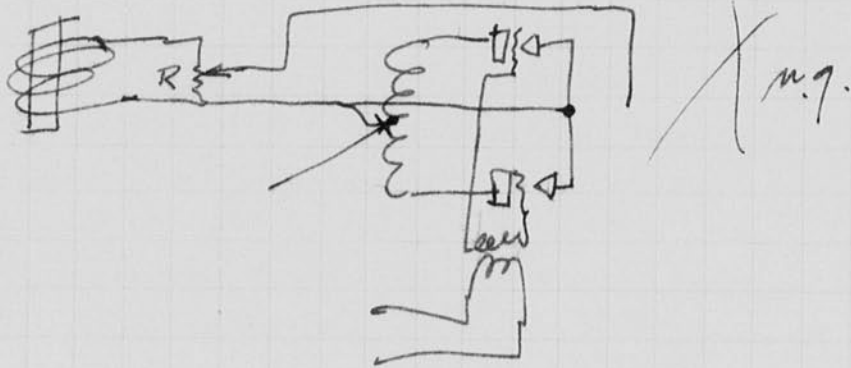
Scale with slots for the light to shine through.



Example
 16 mm film
 driven at constant speed by a small synchronous motor so that there would be 60 frames per second. A relay system would start the camera and the stroboscope simultaneously.

May 29/1931. Field-Switching for
 Alternating Synchronous Motors

A thyatron supply on the field of a synchronous motor can be made to supply intermittent field current so that the field is on from 0 to 180 degrees and is off from 180 to 360 (generator angles).



May 30, 1931
H. S. Edgerton

Phase angle control of thyatrons.

Ref. W. B. Nottingham. March 1931 p 271.
"Characteristics of small grid-controlled
hot-cathode mercury arcs or thyatrons"

It shows a 24 degree (60 cycle) phase
shift in curves for an F. & 27 and a
FG 17. The circuit that he uses depends
upon a condense and a resistance
for the phase shifting. The angle
is calculated since R and C are
known. This shift in phase corresponds
to 1×10^{-3} seconds or 1000 micro sec.
which is a very long time.

I am sure that this question needs
further investigation. It may be
possible that the current in the
grid circuit makes his calculation
of phase angle invalid especially
since the R and C must be in a
steady state in order to compute
the angle by the impedance method.

if $\alpha = 45^\circ$ $R = 1 \times 10^6$ ohms.
 $X_c = \frac{1}{\omega C} = 1 \times 10^6$ ohms.

time const $\tau = RC = R \frac{L}{\omega X_c} = \frac{1 \times 10^{16}}{377 \times 10^6} = \frac{1}{377}$ sec,
of phase
shifting circuit.
 $= .00265$

Another factor which may make his
angle calculations invalid are currents
between the anode and grid before
conduction. I have noticed such a
glow between anode and grid
in the long arm glass thyatrons before
the main arc strikes.

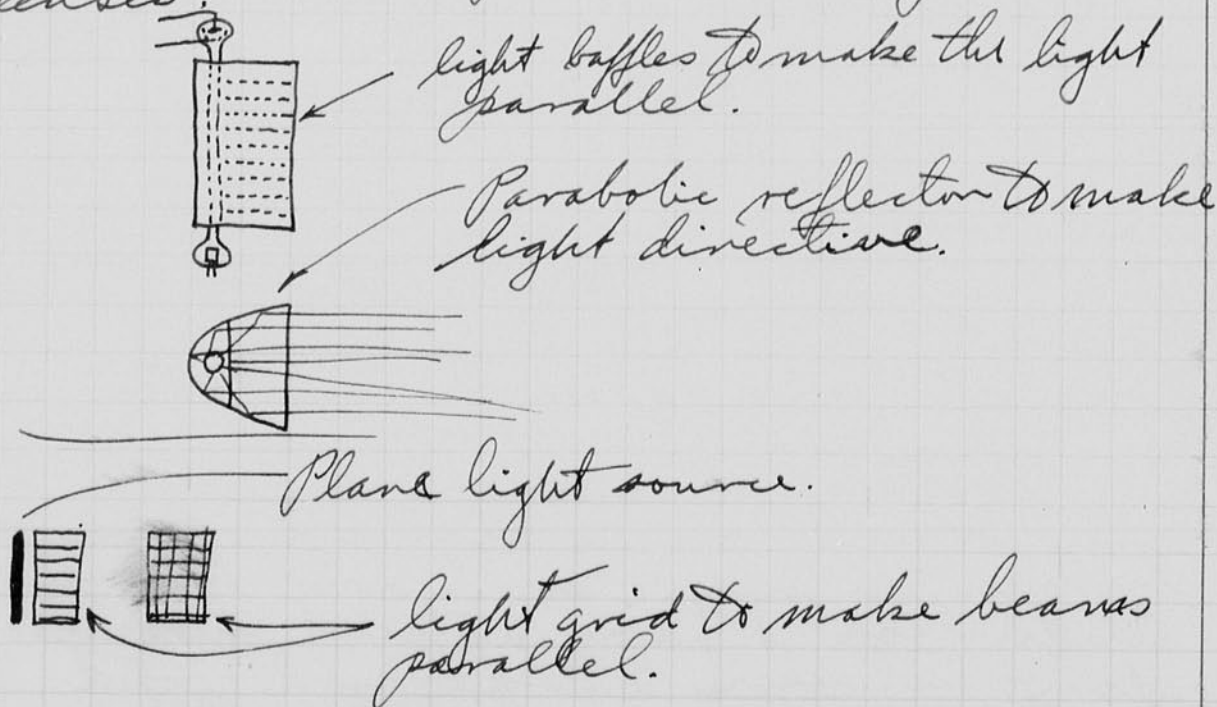
June 16, 1931.
H.E. Edgerton

Photograph

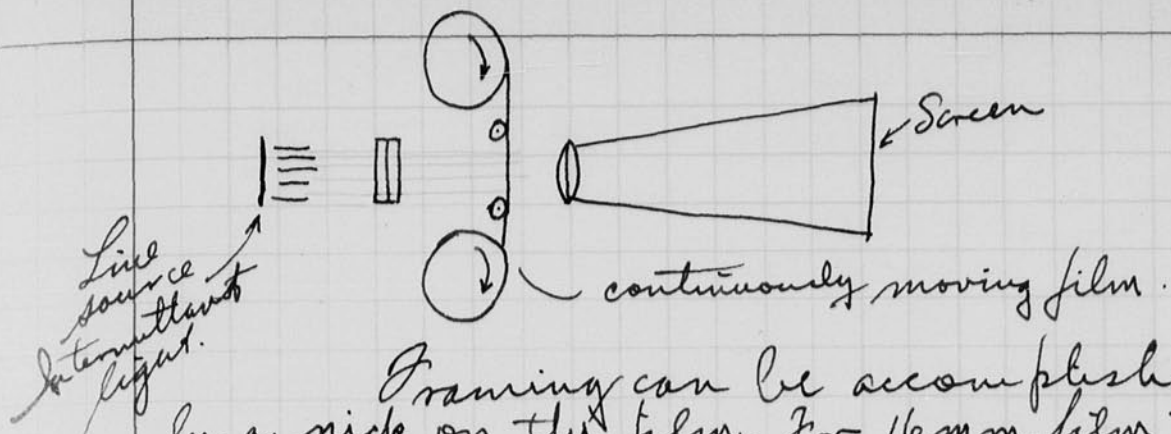
Movie Projection by means
of intermittent light.

Practically all present
projection schemes use a
concentrated light source
which approaches a point
source.

Such a point source appears
at the present to be impossible
with the mercury arc lamp.
It may be possible however
to use a line source of light
to advantage. I believe that an
intense capillary can be
built which will be small in
diameter but very intense. A
parabolic mirror and parallel
light shields may make it
feasible to get a parallel beam
of light which may be directed
and focused by means of cylindrical
lenses.



During the last week I worked at the Sprague
Specialties company at Quincy on the motor driving the
Visivox. These intermittent light schemes were discussed
at length with Bill Dunn. constant speed film seems to have advantages



Framing can be accomplished by a nick on the film. For 16 mm film the sprocket holes will accomplish this purpose.

Notebook # 3

Filming and Separation Record

___ unmounted photograph(s)

___ negative strip(s)

1 unmounted page(s)
(notes, drawings, letters, etc.)

was/were filmed where originally located between page ___ and ___.
in side back cover

Item(s) now housed in accompanying folder.

Oscillation data for determination of WR²

I_f 804 ^A	$V \phi$	$E_a \phi$	X_c	time in Secs	Cycles of oscill.	WR ²	
4.0	$120/\sqrt{3}$	68.5	3.26	34 23	15 10	323	} From 187 KVA machine. iron core reactors
8.8	$126.5/\sqrt{3}$	140	"				
8.7	$124/\sqrt{3}$	139	"	32 ? 29.75	20		
8.7	$123/\sqrt{3}$	139	"	30.0		294	
5.92	$120/\sqrt{3}$	101	"	36.5	20	309	
4.15	$118/\sqrt{3}$	63.5	"	16 ? 17.5	15	278	
"	"	"	"	18.25	15		

Iron core removed from reactors.

4.15	$115/\sqrt{3}$	73	1.41	19.5	15	252	} From 187 KVA machine
"	"	73	1.41	20	15	266	
8.3	$231/\sqrt{3}$	134	1.41	20.25	30	250	
<u>machine operating from bus.</u>							
2.9	$226/\sqrt{3}$	50	1.09	13.5	15	210	

Handwritten scribble or signature in the top right corner.

H. E. E.